UNIVERSITY OF AGDER

Maximizing the Diversification Ratio in the Norwegian stock market

A portfolio approach in the period 2000-2012

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This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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PREFACE

This thesis marks the end of a two – year Master's degree in Economics and Business Administration with specialization in financial economics. The purpose is to apply scientific methods to explore specific research questions. It has been an instructive journey with many challenges along the way.

This process has given me the opportunity for a deeper insight of portfolio theory, which I find as an interesting topic. In addition, I had to learn the statistical software program R to conduct the analysis. This has perhaps been the greatest challenge, since I was new to the programming-environment. However, learning this software has opened many doors to how I could expand my analysis and thereby increasing my interest for portfolio theory even further.

I would like to thank my supervisor, Trygve Kastberg Nilssen, for steady guidance and helpful inputs. Also, I would like to thank my family for support and motivation during this semester.

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ABSTRACT

This analysis is based on the article by Choueifaty & Coignard (2008) where a "most-diversified portfolio" (MDP) is compared to a market capitalization – weighted benchmark, a global minimum variance portfolio and an equally-weighted portfolio. The weight allocation of assets in a MDP is determined by maximizing the diversification ratio. They show that the MDP outperforms the other portfolios in terms of higher diversification ratio and higher Sharpe ratio. In this analysis I determine if similar results can be achieved in the Norwegian stock market, for the MDP. I focus on the portfolio performance and the portfolio diversification, although these two terms coincide to some degree. The annualized Sharpe ratio measures the performances, while the diversification ratio, rolling volatility and correlations are used to measure diversification. I create two MDPs. Portfolio number one has no short-sale restrictions. Portfolio number two has a long-only constraint. The performances of the MDPs are compared to each other, a cap-weighted benchmark and an equally-weighted portfolio in the period 2000-2012. I show that the MDPs have higher Sharpe ratios and are more diversified than the cap-weighted portfolio and the equally-weighted portfolio, when we look at the total period. However, when the annualized Sharpe ratio is calculated each year, the MDPs deliver substantially lower Sharpe ratios in 2001 and 2008-2012 compared to the cap-weighted benchmark and the equally-weighted portfolio. All four portfolios show lower Sharpe ratios in these periods due to the dot-com bubble that burst in the beginning of 2000, and the financial crisis that hit the global economy in 2008. In the period 2000-2012, the MDPs deliver lower Sharpe ratios in recessions and superior Sharpe ratios in economic upturns.

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CHAPTER 1 – INTRODUCTION

There are different views regarding the profitability of active portfolio management. This originates from the view of market portfolio efficiency. From Markowitz and modern portfolio theory we have learned that if two portfolios have the same expected return but portfolio number one has a lower volatility, then portfolio number two is inefficient. The same holds if the volatilities are the same but the first portfolio has a higher expected return. If a portfolio generates higher expected return than the market, then this portfolio should also have greater risk relative to the market index, given that the market index is defined as efficient.

In this analysis I want to focus on two issues. Inspired by the research of Choueifaty & Coignard (2008), I want to adopt this to the Norwegian stock market. I chose sixty-five stocks from the Oslo Stock Exchange as the stock universe. The goal is to explore the performance of the MDP compared to a cap-weighted portfolio and an equally-weighted portfolio in terms of Sharpe ratios, in the period 2000-2012. In other words, I want to examine if the MDP is the efficient portfolio in this context.

Next, I want to take a closer look at the diversification of a MDP. This is the main reason why I include an equally-weighted portfolio in the analysis. In theory, by applying equal weights to sixty-five stocks, this portfolio should be well diversified. By comparing the diversification of these four portfolios, this would give a clear indication whether applying portfolio weights by maximizing the diversification ratio, actually leads to a more diversified portfolio.

In the analysis I create two MDPs. The sum of asset weights must equal to one for both portfolios. For one portfolio this is the only constraint, i.e. short sales are allowed. This will be called MDP – short. The other portfolio has in addition a long-only constraint, i.e. all asset weights must be larger than (or equal to) zero. This will be named MDP – long. It is clear that the efficient set of the constrained portfolio must lie within (or on the boundary) of the unconstrained efficient set when the goal is to maximize the diversification ratio. However, I want to examine if the possibility to have short positions, poses a great difference in performance and diversification for this portfolio.

In the next chapter I present a quick summary of portfolio theory followed by an explanation of the diversification ratio. Research regarding efficiency of market – cap weighted portfolios, is presented at the end of this chapter. In chapter three I go through the data and methodology of how I constructed the most-diversified portfolios, and the procedure behind shrinking the covariance matrix. Chapter four gives a mathematical derivation of the diversification ratio which leads to an analytic solution of the portfolio weights. This is followed by a discussion of how well the diversification ratio captures the actual portfolio diversification. Chapter five compares the two most-diversified portfolios to the capitalization-weighted benchmark and the equally-weighted portfolio; in terms of performance and diversification. Finally, the conclusion of my analysis is found in chapter six.

CHAPTER 2 THEORY

2.1 A quick review of portfolio theory

2.1.1 Basics

The modern portfolio theory is over sixty years old. It began with the article published by Markowitz (1952) where he introduced the concept of how portfolio risk can be reduced by not putting all eggs in one basket. Although this concept existed long before this article was published, the formal portfolio construction where this is accounted for was new to the finance world (Bodie, Kane & Marcus, 2011). A problem with Markowitz' approach has been the high degree of complexity when we include a large number of assets. Due to the fact that there are easier access to advanced software to do these mathematical computations, this issue is not as important as before. In the following paragraph I introduce some basic statistical measurements for the return of the stock price. Next I apply this to a portfolio of *n* stocks and derive a formula for the portfolio return, variance and standard deviation. Then I rewrite these formulas as vectors and matrices, and derive a result from matrix derivation that is used later in the thesis.

Consider an investor who buys stock A at time t for the price P_t, and sells the stock at time t+1 for P_{t+1}. The simple realized return (r) for the investor at t+1, is (Berk & DeMarzo, 2011, p. 298):

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t} = \frac{P_{t+1}}{P_t} - 1. \tag{1}$$

We now assume that the investor did not sell the stock and observes the return over a specific period. The price at t+1 is uncertain at time t and we assume that the return is an independent and identically distributed random variable. The investor can calculate the expected return, $\mathbb{E}(r)$, of the stock based on the historical data (Bodie et. al, 2011, p. 158):

$$\mathbb{E}(r) = \frac{1}{T}\sum_{t=1}^{T} r_t\,, \tag{2}$$

which is the arithmetic average of the simple returns. Note that by dividing by *T*, we use equal probabilities for each observation. It would be interesting for the investor to know how much the realized returns deviates from the expected value. The variance of return is equal to the expected value of squared deviations from the mean. Since we use historical returns we calculate the deviations from the simple arithmetic average (2). The true expected return is unobservable, so by replacing it with the arithmetic average we estimate the variance (Bodie et.al, 2011, p.161):

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mathbb{E}(r))^2. \tag{3}$$

The distance between observed – and expected value is squared so that the variance is non-negative. The purpose of this is to prevent that sums of deviations below and over the expected value reduce the total variance. This expression is divided by *n-1* to eliminate the estimation error that results from using the arithmetic mean and not the true, unknown expected value[1]. In other words, since the simple return of the first observation cannot be calculated, and this term is included in the variance of return, we "loose" one degree of freedom.

The variance can be viewed as a risk measure, but the units are defined in squared returns. It would be difficult to compare the risk of different stocks with this measure. An answer to this problem is to apply the square root to the variance. We are then left with the standard deviation which is measured in the same units as the return (Bodie et. al, 2011, p. 161):

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mathbb{E}(r))^2}. \tag{4}$$

If the investor now in addition buys another stock, we can calculate the covariance of returns of the two stocks, i.e. how the two stock returns move together (Berk et.al., 2011, p. 333):

[1] The arithmetic average of squared deviations is multiplied by n/(n-1), but since the *n* in the numerator in this term and the *n* in the denominator of the second term cancel out, we are left with this expression; $\frac{n}{n-1}\cdot\frac{1}{n}=\frac{1}{n-1}$

$$Cov(r_i, r_j) = \frac{1}{T-1}\sum_{t=1}^{T}\big(r_{i,t} - \mathbb{E}(r_i)\big)\cdot(r_{j,t} - \mathbb{E}(r_j)). \qquad (5)$$

From (5) we see that if the two stocks move in the same direction (both returns move below or above the mean at the same time), the covariance is positive. If they move in opposite direction the covariance is negative. If the two returns are stochastically independent then the covariance is zero[2]. We divide by T-1 since we use historical data, which makes (5) an estimated covariance.

To quantify the strength of the relationship between the two returns we can calculate the correlation between them (Berk et.al, 2011, p. 334):

$$Corr(r_i, r_j) = \frac{Cov(r_i, r_j)}{\sigma_i \sigma_j}. \qquad (6)$$

Since we divide the covariance by the volatility of the two returns, the correlation coefficient will be a number between -1 and +1. The sign of the correlation follows the same logic as with the covariance.

2.1.2 Portfolio of *n* stocks

Now consider *n* risky stocks where r_i is the return (*i* represents any of the *n* stocks). We assume that r_i is an independent and identically distributed random variable. If we construct a portfolio of these stocks, then the portfolio return is equal to the weighted sum of the returns (Berk et. al, 2011, p. 331):

$$r_p = \sum_{i=1}^{n} w_i r_i \, , \qquad (7)$$

where w_i denotes the weight of asset *i*. The expected return for each stock is $\mathbb{E}(r_i)$. The portfolio expected return can be written as (Bodie et. al, 2011, p. 241):

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[2] Note that this does not necessarily mean that if Cov(r_i, r_j) = 0 the two stocks are stochastically independent.

$$\mathbb{E}(r_p) = \sum_{i=1}^{n} w_i \mathbb{E}(r_i). \tag{8}$$

The portfolio variance is the sum of the variance for each stock return and the pairwise covariance[3] between them:

$$Var(r_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \, Cov(r_i, r_j). \tag{9}$$

We define the portfolio variance by σ_p^2 and $Cov(r_i, r_j)$ as σ_{ij}. Rewriting (9) as (Bodie et.al, 2011, p. 241):

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}. \tag{10}$$

From (10) we get the portfolio standard deviation by applying the square root to the portfolio variance:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}}. \tag{11}$$

The portfolio standard deviation is a widely used measure of portfolio risk.

2.1.3 Portfolio calculation in matrix notation

Before introducing equation (7), (8), (10) and (11) in matrix notation, it is necessary with some basic definitions.

Every matrix and vector will be denoted in bold text. We define:

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[3] This is a simplified expression since we do not assume that i is different from j. Since Cov(r_i, r_i) = Var(r_i) the weighted sum of variances is captured by (9).

$$\mathbf{w}_{n,1} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \quad , \qquad \mathbf{R}_{n,1} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} \text{ and } \quad \boldsymbol{\mu}_{n,1} = \begin{pmatrix} \mathbb{E}(r_1) \\ \mathbb{E}(r_2) \\ \vdots \\ \mathbb{E}(r_n) \end{pmatrix}$$

where **w** is a vector of portfolio weights, **R** is a vector of stock returns and **μ** is a vector of expected returns. The subscript of each vector (and matrices) defines the dimension; for example $\mathbf{w}_{n,1}$ means that the portfolio weights contain *n* rows and 1 column. This is also called a column vector. By using basic matrix calculation we see that (7) can be written as (Markowitz, 1959, p. 172):

$$\sum_{i=1}^{n} w_i r_i = (w_1 \quad w_2 \quad \cdots \quad w_n) \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} = \mathbf{w}'\mathbf{R} = r_p. \tag{12}$$

The expected return of the portfolio can be written in the same manner:

$$\sum_{i=1}^{n} w_i \mathbb{E}(r_i) = (w_1 \quad w_2 \quad \cdots \quad w_n) \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} = \mathbf{w}'\boldsymbol{\mu} = \mathbb{E}(r_p), \tag{13}$$

where $\mathbf{w}'$ is the transpose of the weight vector, meaning that the weight in the *i*'th row in the *j*'th column[4] is placed in the *j*'th row in the *i*'th column. We use the transpose to be able to calculate the inner product. In general, matrix multiplication is only possible if number of columns in the first matrix is equal to the number of rows in the second matrix.

The portfolio variance is expressed as:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = (w_1 \quad w_2 \quad \cdots \quad w_n) \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{n1} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = \boldsymbol{w}'\boldsymbol{\Omega}\,\boldsymbol{w} = \sigma_p^2, \tag{14}$$

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[4] In this case j=1.

where $\boldsymbol{\Omega_{n,n}}$ is the covariance matrix[5]. This is a symmetric matrix ($\sigma_{ij} = \sigma_{ji}$) with the variances along the diagonal. We multiply the covariance matrix with $\mathbf{w}$ and $\mathbf{w}'$ to get the weights squared. To calculate the portfolio standard deviation we simply apply the square root to (14):

$$\sqrt{\boldsymbol{w}'_{1,n}\boldsymbol{\Omega_{n,n}}\boldsymbol{w_{n,1}}} = \sigma_{\boldsymbol{p}}. \tag{15}$$

Later in the thesis, when I create the most-diversified portfolios, I calculate the derivative of the diversification ratio with respect to **w**. In this section I will explain how to calculate the first order conditions of the portfolio variance.

The portfolio variance is written as $\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}$. It is easier to illustrate the derivative of this function w.r.t **w** by using a simple example:

Let $\mathbf{w}$ be a column vector with two rows:

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}.$$

$\boldsymbol{\Omega}$ is a symmetric matrix. Since **w** has two rows the dimension of this matrix is 2 by 2:

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

Since $\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = \mathrm{a}w_1^2 + 2\mathrm{b}w_1w_2 + \mathrm{c}w_2^2$ we get (Zivot, 2011):

$$\frac{\partial\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}{\partial\mathbf{w}} = \begin{pmatrix} \dfrac{\partial\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}{\partial w_1} \\ \dfrac{\partial\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}{\partial w_2} \end{pmatrix} = \begin{pmatrix} 2aw_1 + 2bw_2 \\ 2bw_1 + 2cw_2 \end{pmatrix}.$$

We see that this can be written in matrix form as follows:

$$2\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = 2\boldsymbol{\Omega}\mathbf{w}. \tag{17}$$

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[5] $\boldsymbol{\Omega_{n,n}}$ = E(($\mathbf{R}_{n,1} - \boldsymbol{\mu}_{n,1}$)($\mathbf{R}_{n,1} - \boldsymbol{\mu}_{n,1}$)') and is a n by n matrix since **R** has n elements (Zivot, 2011)

2.2 Diversification ratio

As mentioned in the previous section, the portfolio standard deviation of returns is used to measure portfolio risk. We distinguish between two sources of risk. On the one hand there is risk associated with the general economy (inflation, interest rates etc.). This is called systematic risk. On the other hand we have risk that is firm-specific, often called unsystematic risk. The difference between them is that the systematic risk affects all firms, but the unsystematic risk only affects the specific firm. In a portfolio context we can reduce the unsystematic risk by including more stocks in the portfolio. Eventually we are left with only the systematic risk. A lot of research has been devoted to finding the number of stocks that make a diversified portfolio. For example in the article by Statman (1987), the result was 30 stocks for a borrowing investor and 40 stocks for a lending investor. In the following paragraph, I illustrate how a stock contributes to portfolio risk by decomposing the variance of portfolio return. Next, I introduce the measurement developed by Choueifaty & Coignard (2008) called the diversification ratio.

An alternative to (10) is to express the variance of portfolio return as the sum of the weighted covariance of stock *i* with the portfolio (Berk et.al, 2011, p. 341):

$$\sigma_p^2 = \sum_{i=1}^{n} w_i \sigma_{ip}. \tag{18}$$

Since $\sigma_{ip} = \sigma_p \sigma_i Corr(r_i, r_p)$ we can rewrite (18) as:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i \sigma_p \sigma_i Corr(r_i, r_p). \tag{19}$$

We divide by $\sigma_p$on both sides. The result is a decomposition of the portfolio volatility:

$$\sigma_p = \sum_{i=1}^{n} w_i \sigma_i Corr(r_i, r_p). \tag{20}$$

From (20) we see that the portfolio volatility depends on the weighted average volatility of each stock, and the correlation between the stock and the portfolio. The correlation adjusts for the risk that is common to the portfolio and stock i. The portfolio volatility will be lower than the weighted average volatility of each stock, unless all stocks are perfect positively correlated with the portfolio ($Corr(r_i, r_p) = 1$).

A way to quantify the degree of diversification is by calculating the diversification ratio (DR). This ratio is defined as:

$$DR(w) = \frac{\sum_{i=1}^{n} w_i \sigma_i}{\sigma_p}. \tag{21}$$

In matrix notation (21) becomes[6]:

$$DR(\mathbf{w}) = \frac{\mathbf{w}'\boldsymbol{\sigma}}{\sqrt{\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}} \tag{22}$$

where $\boldsymbol{\sigma}$ is a column vector of the volatility of each stock:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{pmatrix}.$$

From (20) we see that the nominator in (22) is the portfolio volatility when every stock is perfectly correlated with the portfolio. The denominator is the actual portfolio risk. We see that the diversification ratio of a portfolio containing one stock is equal to one. In reality, every stock will not be perfectly correlated with the portfolio, so the diversification ratio will be larger than one when the portfolio consists of more than one stock (Sapra, 2011). By maximizing this ratio subject to constraints, we get the most diversified portfolio. This is done in the next chapter.

[6] The matrix dimensions are defined in chapter 2.1.3, and will not be included further in the thesis.

The maximization of the diversification ratio is a different approach when it comes to diversifying a portfolio. Instead of increasing number of stocks in the current portfolio, we rearrange the weights of the existing stocks to achieve the highest possible diversification ratio. In essence: higher diversification ratio → a more diversified portfolio.

2.3 Is the cap-weighted portfolio efficient?

The study of the efficiency of investing in a market cap-weighted portfolio is not a new subject. Haugen & Baker (1991) showed that cap-weighted stock portfolios are inefficient, except under extremely restrictive conditions. They argued that the theory predicts this portfolio to be inefficient even under the assumption that the capital market is "informationally" efficient.

Amenc, Goltz, Martellini & Retkowsky (2010) also recognized this inefficiency and, by focusing on the tangency portfolio, developed a procedure for generating efficient indices with Sharpe ratios considerably higher than the cap-weighted index.

Chow, Hsu, Kalesnik & Little (2011) developed alternative passive investment strategies that outperformed cap-weighted indexing, unadjusted for risk factors. These strategies were categorized by heuristic-based weighting methodologies and optimization-based weighting methodologies.

Arnott, Hsu & Moore (2005) compared the cap-weighted index to fundamental equity market indices, i.e. weights were measured by fundamental factors other than market capitalization. They showed that these index portfolios delivered superior mean-variance performances.

Lee (2011) argues that there is no theory to predict the performance of risk based portfolios relative to the market. For some active portfolios to outperform the market portfolio, others have to underperform. If a portfolio consistently outperforms the market, then it has more information on future asset returns than the market portfolio.

CHAPTER 3 METHODOLOGY

3.1 Data and strategy

I include 65 stocks from the Norwegian stock market as the stock universe in this analysis. The firms are listed in appendix A. I use daily data from 01.01.1999 to 01.01.2013. This is downloaded from the Thomson Reuters Datastream. To create the most-diversified portfolios, the necessary inputs are the (inverted) covariance matrix and the volatility vector. All inputs are based on historical data of 250 days. To improve the estimated covariance matrix, and to avoid problems regarding singularity, I applied a shrinkage method to the covariance matrix. From this I add the resulting weights to the stock returns. This rebalancing is done on a quarterly basis from January 2000 to December 2012.

To conduct the analysis, I use the statistical software program R. Packages that came to good use are listed in the credits. I created a function in R to calculate the MDPs. This function is listed in appendix C. The benchmark is constructed by applying weights according to the market capitalization of the 65 firms. In this way the benchmark and the most-diversified portfolios are comparable since they have the same universe of stocks. The benchmark is also rebalanced on a quarterly basis. This benchmark is however not a market cap-weighted index, as opposed to the benchmark used in the article by Choueifaty & Coignard (2008). OBX is a tradable index and includes the 25 most liquid stocks on the Oslo Stock Exchange. The firms are weighted according to their market capitalization. The index is rebalanced on a semi-annual basis. I chose not to use this index as a benchmark for two reasons. First, it is difficult to obtain data from the time period I want to examine. Reason number two is that this index contains fewer stocks than I would like in the portfolios, since a significant part of the analysis deals with portfolio diversification. I am mainly interested in comparing portfolios with different allocations of weights. However, the correlation of returns between the benchmark and OSEBX is 0.95, and 0.94 between the benchmark and OBX. In other words, this benchmark is a valid representation of market cap-weighted index returns.

The equally-weighted portfolio contains approximately 1.54% of every stock.

3.2 Shrinkage estimator

The sample covariance matrix is an unbiased estimator, however when number of observations (*t*) is less than the number of stocks (*n*) it would be a problem inverting the covariance matrix. Also when *t* is close to *n*, estimation errors may occur. Ledoit & Wolf (2004) points out that an estimator with a lot of structure will handle this problem, but such estimators tend to be biased. They state that all successful risk models contain a compromise between an unstructured and a highly structured estimator. By this philosophy they develop a linear convex combination between the sample covariance matrix, i.e. unstructured estimator, and a highly structured estimator (T):

$$\boldsymbol{\Omega}^* = \lambda \mathbf{T} + (1 - \lambda)\boldsymbol{\Omega} \tag{23}$$

where λ is a constant[7]. We see that **Ω** is "shrunk" towards T. This is the technique called shrinkage. There are different ways of shrinking the covariance matrix. The method used in this analysis is based on Schäfer & Strimmer (2005), which can be used in R by installing the package *corpcor*. Here, the estimated covariance matrix is determined by shrinking the variance of each stock and the correlation matrix. In order to illustrate this we define **V** as a matrix with *n* rows and *n* columns, with the variances along the diagonal:

$$\mathbf{V} = \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n^2 \end{pmatrix}.$$

This gives us:

$$\widetilde{\boldsymbol{\sigma}} = \sqrt{\mathbf{V}}$$

where $\widetilde{\boldsymbol{\sigma}}$ is a matrix with *n* rows and *n* columns with the volatilities along the diagonal. The covariance matrix can now be expressed as:

$$\boldsymbol{\Omega} = \widetilde{\boldsymbol{\sigma}}\mathbf{C}\widetilde{\boldsymbol{\sigma}}$$

where **C** is the correlation matrix. By shrinking the variance and the correlation matrix, we get the following expression for the estimated covariance matrix:

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[7] Also called the "shrinkage-intensity".

$$\mathbf{\Omega}^* = \widetilde{\boldsymbol{\sigma}}^* \mathbf{C}^* \widetilde{\boldsymbol{\sigma}}^* \tag{24}$$

where $\widetilde{\boldsymbol{\sigma}}^* = \sqrt{\mathbf{V}^*}$. These expressions are explained in more detail in the next two sub-chapters.

3.2.1 The variance shrinkage intensity

By following the same logic as with (23) we can write the estimated variance (**V***) as:

$$\mathbf{V}^* = \hat{\lambda}_v^* v_{median} + \left(1 - \hat{\lambda}_v^*\right)\mathbf{V},$$

where v_{median} is the median of the variances. The median is, in this case, the target estimator.

The shrinkage intensity for the variance can be expressed as (Opgen-Rhein & Strimmer, 2007):

$$\hat{\lambda}_v^* = min(1, \frac{\sum_{i=1}^{n} \widehat{Var}(v_i)}{\sum_{i=1}^{n} (v_i - v_{median})^2}).$$

We see that if the empirical variance deviates a lot from the median (the target), then there will be less shrinkage. The *min( )* function is used to avoid over – and under shrinkage. In other words, the shrinkage intensity can only be a number between 0 and 1.

The expression for $\widehat{Var}(v_i)$ and v_i is straightforward. From (2) we have the arithmetic average of simple returns which, for simplicity, we will define here as $\bar{r}$. We define:

$$m_{t,i} = (r_{t,i} - \bar{r})^2 \; and$$

$$\bar{m}_i = \frac{1}{T} \sum_{t=1}^{T} m_{t,i}.$$

From this we get:

$$v_i = \frac{T}{T-1} \bar{m}_i \; and$$

$$\widehat{Var}(v_i) = \frac{T}{(T-1)^3}\sum_{t=1}^{T}(m_{t,i} - \bar{m}_i)^2.$$

3.2.2 The correlation shrinkage intensity

When estimating the correlation matrix, we shrink the empirical correlation towards the identity matrix:

$$\mathbf{C}^* = \hat{\lambda}_C^*\mathbf{I} + \left(1 - \hat{\lambda}_C^*\right)\mathbf{C},$$

where (Schäfer & Strimmer, 2005):

$$\hat{\lambda}_C^* = min\left(1, \frac{\sum_{i \neq j}\widehat{Var}\left(r_{ij}\right)}{\sum_{i \neq j} r_{ij}^2}\right).$$

r_{ij} denotes the elements in the empirical correlation matrix. We see that if the correlations are high, then less shrinkage will be applied towards the identity matrix. The expression for $\widehat{Var}\left(r_{ij}\right)$ is derived in a similar fashion as in the previous sub-chapter. From (6) we know that the measures need to be standardized, i.e. $r_{ij} = \frac{\sigma_{ij}}{\sigma_i\sigma_j}$. From this we get that:

$$m_{t,ij} = \frac{1}{\sigma_i\sigma_j}\left(r_{t,i} - \bar{r}_i\right)\cdot\left(r_{t,j} - \bar{r}_j\right).$$

Now, we define

$$\bar{m}_{ij} = \frac{1}{T}\sum_{t=1}^{T} m_{t,ij}.$$

The estimated variance of the correlation elements can now be written as:

$$\widehat{Var}\left(r_{ij}\right) = \frac{T}{(T-1)^3}\sum_{t=1}^{T}(m_{t,ij} - \bar{m}_{ij})^2.$$

A simple numerical example to illustrate this is given at the end of Appendix C.

CHAPTER 4 THE MOST-DIVERSIFIED PORTFOLIO

4.1 Maximizing the diversification ratio

The strategy of determining weights by maximizing the diversification ratio is defined as an anti-benchmark strategy (Choueifaty, 2006). There are however different anti-benchmark strategies that leads to a most-diversified portfolio. The first approach is the one used in this thesis. The analytical solution (30.1) below is not explicitly stated in the article. Due to the fact that the diversification ratio is similar to the Sharpe ratio, I used the procedure for maximizing this ratio (Zivot, 2011 and Blake, 2011) as a benchmark. To ensure that this solution is correct, I used the Solver in Microsoft Excel to calculate the first set of weights for the MDP – short and compared to the weights retrieved from (34.2).

The optimization problem for the MDP – short can be formulated as (Choueifaty, Froidure & Reynier, 2011):

$$\min_{\mathbf{w}} \frac{1}{2}\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \;\; s.t. \;\; \mathbf{w}'\boldsymbol{\sigma} = 1. \tag{25}$$

We multiply by $\frac{1}{2}$ to be able to simplify the first order condition. This method can be applied since *DR(*$\mathbf{w}$*)* is homogenous of degree zero:

$$DR(2\mathbf{w}) = \frac{2\mathbf{w}'\boldsymbol{\sigma}}{\sqrt{2\mathbf{w}'\boldsymbol{\Omega}2\mathbf{w}}} = \frac{\mathbf{w}'\boldsymbol{\sigma}}{\sqrt{\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}} .$$

In other words, although we scale the function by $\frac{1}{2}$, we will still have the same solution because of this property.

This optimization problem has the following Lagrange function (Pemberton & Rau, 2007, p. 321):

$$\mathrm{L}(\mathbf{w}, \lambda) = \frac{1}{2}\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} - \lambda(\mathbf{w}'\boldsymbol{\sigma} - 1). \tag{26}$$

First order conditions:

$$\frac{\partial L(\mathbf{w},\lambda)}{\partial \mathbf{w}} = \frac{1}{2} \cdot 2\boldsymbol{\Omega}\mathbf{w} - \lambda\boldsymbol{\sigma} \; = 0 \tag{27}$$

$$\frac{\partial L(\mathbf{w},\lambda)}{\partial \lambda} = \mathbf{w}'\boldsymbol{\sigma} - 1 = 0. \tag{28}$$

(27) is a result from the derivation rule derived at the end of chapter two. We rewrite and simplify this expression as:

$$\boldsymbol{\Omega}\mathbf{w} = \lambda\boldsymbol{\sigma}.$$

Solve with respect to **w**:

$$\mathbf{w} = \lambda\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}. \tag{29}$$

(28) can be written as:

$$\mathbf{w}'\boldsymbol{\sigma} = 1.$$

Since $\mathbf{w}'\boldsymbol{\sigma} = \boldsymbol{\sigma}'\mathbf{w}$ we get:

$$\boldsymbol{\sigma}'\mathbf{w} = 1.$$

By multiplying $\boldsymbol{\sigma}'$ on both sides of (29), we get:

$$\boldsymbol{\sigma}'\mathbf{w} = \lambda\boldsymbol{\sigma}'\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma} = 1.$$

Now we find an expression for λ[8]:

$$\lambda = \frac{1}{\boldsymbol{\sigma}'\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}} \; .$$

Finally, we insert this expression in (29):

$$\mathbf{w} = \frac{\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}}{\boldsymbol{\sigma}'\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}} \; . \tag{30.1}$$

[8] Since $\boldsymbol{\sigma}'\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}$ is just a number (as opposed to a vector or matrix), we can divide it on both sides. For vectors and matrices, we need to multiply both sides by its inverse.

The portfolio weights depend on the inverted covariance matrix and the volatility vector. Stocks with higher (lower) volatility will have smaller (larger) weights in the portfolio. In addition we want the portfolio weights to sum to one, so we need to rescale the weights.

$$\mathbf{w}^* = \begin{pmatrix} \frac{w_1}{\sum_{i=1}^{n} w_i} \\ \frac{w_2}{\sum_{i=1}^{n} w_i} \\ \vdots \\ \frac{w_n}{\sum_{i=1}^{n} w_i} \end{pmatrix}. \tag{30.2}$$

For a portfolio with a positive weights constraint, the maximization problem can be defined as:

$$\min_{\mathbf{w}} \frac{1}{2}\mathbf{w}'\mathbf{\Omega}\mathbf{w} \quad s.t. \qquad \begin{array}{l} \mathbf{w}'\boldsymbol{\sigma} = 1 \\ w_i \geq 0 \end{array} \tag{31}$$

where *i* represents the weight of a stock in the portfolio. Because of this added constraint we can no longer derive an analytic solution. This maximization problem has to be solved numerically (Zivot, 2011), i.e. we have to try different weight combinations and chose the one that generates the highest diversification ratio. Similar to the optimization above, we need to rescale the weights to sum to one.

As shown by Choueifaty & Coignard (2008), a different approach is to define X_1, X_2, ..., X_n as stocks in universe U. To simplify the math we imagine a universe (U_s) of synthetic stocks (S_1, S_2, ..., S_n), i.e. all stocks have the same expected volatility. In mathematical terms a synthetic stock is defined as[9]:

$$S_i = \frac{X_i}{\sigma_i}$$

where σ_i is the volatility of X_i. For simplicity we assume that the expected volatility of S_i is equal to 1. Since this is a universe of synthetic stocks, this means that all the stocks in this

[9] For simplicity I do not include the risk-free investment.

universe have expected volatilities equal to 1. The volatility vector is therefore a vector of ones:

$$\boldsymbol{\sigma}_s = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

The diversification ratio for the synthetic stocks is defined as:

$$DR(\mathbf{w}_s) = \frac{\mathbf{w'}_s \boldsymbol{\sigma}_s}{\sqrt{\mathbf{w'}_s \boldsymbol{\Omega}_s \mathbf{w}_s}}$$

where $\mathbf{w}_s$ is the column vector of weights for the portfolio of synthetic stocks, and $\boldsymbol{\Omega}_s$ is the covariance matrix for this portfolio. As with the approach above we want the weights to sum to 1. Since the volatility vector is a vector of ones, this means that the denominator is equal to one.

$$(w_{s1} \quad w_{s2} \quad \cdots \quad w_{sn}) \cdot \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = 1.$$

We can now write the diversification ratio as:

$$DR(\mathbf{w}_s) = \frac{1}{\sqrt{\mathbf{w'}_s \boldsymbol{\Omega}_s \mathbf{w}_s}}.$$

The only way for DR_s to increase is for the denominator to decrease, so maximizing this ratio is equivalent to minimizing the portfolio volatility. Since every stock has an expected volatility of 1, the covariance matrix is equal to the correlation matrix (**C**) of the original stocks (X_i). This approach is therefore a minimization problem and can be defined as:

$$\min_{\mathbf{w}} \mathbf{w'}_s \mathbf{C} \mathbf{w}_s \quad s.t \quad \mathbf{w'1} = 1. \tag{32}$$

We can of course add more constraints to (32). The solution from this optimization is however not the final result. These weights are based on a universe of synthetic stocks. We need to construct a real portfolio (M) by dividing each optimal weight (w_{Si}^*) by the actual volatility (σ_i)[10]:

$$\boldsymbol{M} = \begin{pmatrix} \frac{w_{S1}^*}{\sigma_1} \\ \frac{w_{S2}^*}{\sigma_2} \\ \vdots \\ \frac{w_{Sn}^*}{\sigma_n} \end{pmatrix}.$$

M is a column vector of portfolio weights and can be defined as the most-diversified portfolio. These weights does not necessarily sum to 1. We would add the remaining weights to the risk-free investment. Since we do not include a risk-free asset, the weights have to be rescaled such that the sum of weights equals 1.

Even though these are two different approaches to a most-diversified portfolio, they have the same solution. This proof is shown in appendix B.

4.2 Diversification ratio as a measure of portfolio diversification

4.2.1 Portfolio volatility vs. DR

As mentioned earlier, the diversification ratio is one way of quantifying the degree of portfolio diversification. However the difference between $\mathbf{w}'\boldsymbol{\sigma}$ and $\sqrt{\mathbf{w}'\boldsymbol{\Omega}\mathbf{w}}$ is a differential diversification measure and not an absolute measure (Meucci, 2010). In other words, there is no absolute classification of a high or low diversification ratio. I use this ratio to compare the four portfolios and can only determine that one diversification ratio is higher relative to another. To some degree we can classify a ratio close to 1 as small, since a portfolio of one stock will have a diversification ratio equal to one. It is clear that a one-stock portfolio is not well diversified. From (21) we see that in order for the ratio to increase, either the portfolio volatility has to decrease or the volatility contribution of each stock has to increase. Since the latter assumes that all stocks are perfectly correlated (which is not a realistic assumption), I want to look at the relationship between the portfolio volatility and the <u>actual</u> diversification ratio in the performance period. For this measure to be an indicator of portfolio diversification

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[10] This procedure is called the Choueifaty Synthetic Asset Back-Transformation.

there should be an inverse relationship between the diversification ratio and the portfolio volatility. In the figures below, the standard deviation of the portfolio is plotted against the actual diversification period for all four portfolios in the performance period. Both measures are calculated at the end of each quarter. The ratio is calculated from January 2000 to December 2012 on a quarterly basis using (21).

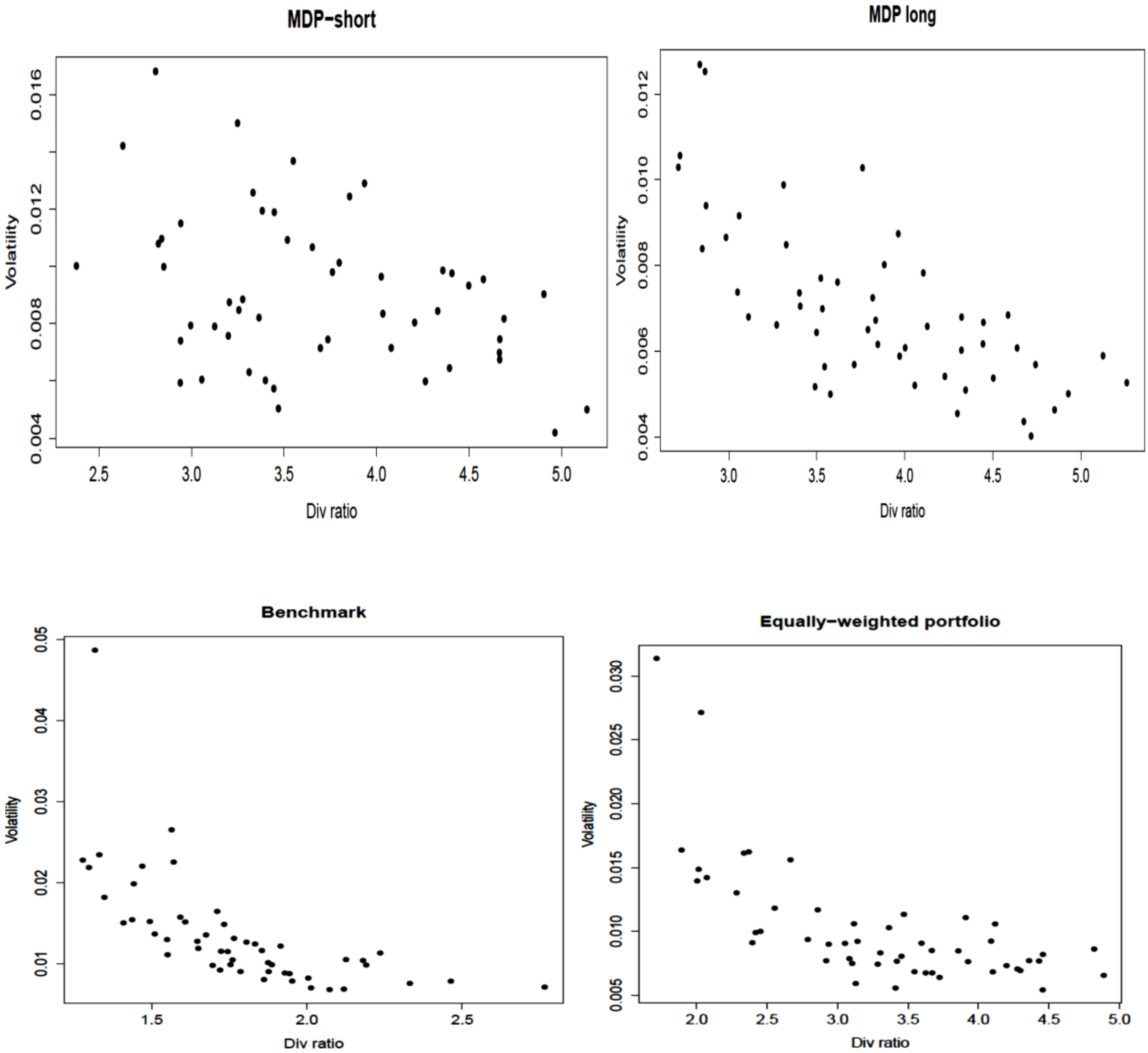

Figure 1: Plots of the portfolio volatility against the actual diversification ratio for all four portfolios in the performance period.

According to Figure 1, it holds for all four portfolios that observations with relatively low volatility have a higher diversification ratio. For observations were the volatility is relatively high, the diversification ratio is lower.

4.2.2 Decomposition of DR

In the article by Choueifaty, Froidure & Reynier (2011), the *DR* is decomposed to:

$$DR(\mathbf{w}) = [\rho(\boldsymbol{w})\big(1 - CR(\boldsymbol{w})\big) + CR(\boldsymbol{w})]^{-\frac{1}{2}}$$

where $\rho(\boldsymbol{w})$ is the volatility weighted average correlation of the stocks in the portfolio:

$$\rho(\boldsymbol{w}) = \frac{\sum_{i \neq j}^{n}(w_i \sigma_i w_j \sigma_j)\rho_{i,j}}{\sum_{i \neq j}^{n} w_i \sigma_i w_j \sigma_j}$$

and $CR(\boldsymbol{w})$ is the volatility weighted concentration ratio

$$CR(\boldsymbol{w}) = \frac{\sum_i^n (w_i \sigma_i)^2}{(\sum_i^n w_i \sigma_i)^2} \ .$$

This shows us that when the concentration ratio decreases and/or the volatility weighted average correlation decreases, the diversification ratio increases. The intuition behind this decomposition is that portfolios with concentrated weights (majority of weights in few stocks) and high correlation between the stocks and the portfolio, are poorly diversified portfolios. This will result in a low diversification ratio.

We can illustrate this decomposition by a simple example. Consider three stocks with the following volatilities, correlations and covariances:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_A \\ \sigma_B \\ \sigma_C \end{pmatrix} = \begin{pmatrix} 0.017 \\ 0.049 \\ 0.018 \end{pmatrix}$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{AB} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{AC} & \sigma_{BC} & \sigma_C^2 \end{pmatrix} = \begin{pmatrix} 0.0002895 & -0.0004813 & 0.0002 \\ -0.0004813 & 0.002456 & 0.00015 \\ 0.0002 & 0.00015 & 0.00033 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \rho_{AA} & \rho_{AB} & \rho_{AC} \\ \rho_{AB} & \rho_{BB} & \rho_{BC} \\ \rho_{AC} & \rho_{BC} & \rho_{CC} \end{pmatrix} = \begin{pmatrix} 1 & -0.571 & 0.644 \\ -0.571 & 1 & 0.166 \\ 0.644 & 0.166 & 1 \end{pmatrix}.$$

To create a most-diversified portfolio of these stocks, we use the approach explained in chapter 4.1. From (30.1) and (30.2) we get the following weights:

$$\mathbf{w} = \begin{pmatrix} w_A \\ w_B \\ w_C \end{pmatrix} = \begin{pmatrix} 1.505 \\ 0.414 \\ -0.919 \end{pmatrix}.$$

By applying (22) we get the diversification ratio:

$$DR(\mathbf{w}) = 3.071.$$

Now we calculate the volatility weighted average correlation of the stocks in the portfolio:

$$\rho(\boldsymbol{w}) = \frac{\sum_{i\neq j}^{n}(w_i\sigma_i w_j\sigma_j)\rho_{i,j}}{\sum_{i\neq j}^{n} w_i\sigma_i w_j\sigma_j}$$

$$= \frac{(1.505\cdot 0.017\cdot 0.414\cdot 0.049)\cdot(-0.571)+(1.505\cdot 0.017\cdot(-0.919)\cdot 0.018)\cdot 0.644+(0.414\cdot 0.049\cdot(-0.919)\cdot 0.018)\cdot 0.166}{(1.505\cdot 0.017\cdot 0.414\cdot 0.049)+(1.505\cdot 0.017\cdot(-0.919)\cdot 0.018)+(0.414\cdot 0.049\cdot(-0.919)\cdot 0.018)}$$

$$= 2.549.$$

Next, we need the volatility weighted concentration ratio

$$CR(\boldsymbol{w}) = \frac{\sum_{i}^{n}(w_i\sigma_i)^2}{(\sum_{i}^{n} w_i\sigma_i)^2}$$

$$= \frac{(1.505\cdot 0.017)^2 + (0.414\cdot 0.049)^2 + ((-0.919)\cdot 0.018)^2}{[(1.505\cdot 0.017) + (0.414\cdot 0.049) + \big((-0.919)\cdot 0.018\big)]^2}$$

$$= 1.577.$$

Finally, we can insert these two numbers in the decomposed DR formula to make sure that the calculations are correct.

$$DR(\mathbf{w}) = \big[\rho(\boldsymbol{w})\big(1 - CR(\boldsymbol{w})\big) + CR(\boldsymbol{w})\big]^{-\frac{1}{2}} = [2.549\cdot(1 - 1.577) + 1.577]^{-\frac{1}{2}} = 3.068.$$

The difference is due to rounding error.

Choueifaty, Y. Froidure, T. & Reynier, J. (2011) also states two core properties of the MDP – long. First, every stock that is not included in the MDP – long (i.e. weight is zero), is more correlated to this portfolio than any of the stocks that are included. In this analysis, the stocks that are included in the MDP – long are on average 1/2 of the whole universe of stocks. However, according to this property this does not mean that the portfolio is poorly diversified. The second core property states that the correlation between the MDP – long and any other long – only portfolio is greater than, or equal to the ratio of its *DR*. The correlations of all assets to the MDP-short are constant, and equal to the inverse of the MDP's *DR*. We can use the example above to illustrate this. The three stocks have the following returns:

$$r_A = \begin{pmatrix} 0.02 \\ 0.01 \\ 0.05 \\ 0.025 \end{pmatrix}, r_B = \begin{pmatrix} 0.01 \\ 0.07 \\ -0.005 \\ 0.1 \end{pmatrix} and\ r_c = \begin{pmatrix} 0.1 \\ 0.09 \\ 0.12 \\ 0.13 \end{pmatrix},$$

i.e. we have four observations of each stock. With the weigh vector, we can calculate the portfolio return:

$$r_p = \begin{pmatrix} -0.0576 \\ -0.0387 \\ -0.0371 \\ -0.0404 \end{pmatrix}.$$

Now we calculate the correlation between each stock and the most-diversified portfolio:

$$\rho_{MDP,A} = 0.3256$$
$$\rho_{MDP,B} = 0.3256$$
$$\rho_{MDP,C} = 0.3256.$$

We see that every stock has a correlation of 32.56% with the portfolio. This is also equal to the inverse of the MDP's diversification ratio:

Note that:

$$DR(\mathbf{w}) = 3.071.$$

When calculating its inverse, we get:

$$DR(\mathbf{w})^{-1} = \frac{1}{3.071} = 0.3256.$$

CHAPTER 5 COMPARING THE FOUR PORTFOLIOS

5.1 Portfolio performances

I measured the performance of the four portfolios in the period 2000-2012. The main focus is on the Sharpe ratios. When analyzing the diversification of the portfolios, I consider the average diversification ratio, the daily observations of the portfolio volatilities and the correlation between the portfolios and each stock.

To compare the portfolios I calculated the annualized return and annualized volatility. The annualized return is calculated by multiplying number of trading days in a year[11] by the average daily return ($\bar{r}$) of the portfolio:

$$R_{\text{annualized}} = \bar{r} * 252. \tag{33}$$

The annualized volatility is calculated in a similar manner. We multiply the average daily volatility ($\bar{\sigma}$) by the square root of number of trading days:

$$\sigma_{\text{annualized}} = \bar{\sigma} * \sqrt{252}\,. \tag{34}$$

By using a risk free rate (r_f) of 4,68% (AF KP) we can calculate the annualized excess return:

$$\begin{aligned} R_{\text{excess}} &= R_{\text{annualized}} - r_f \\ &= R_{\text{annualized}} - 0.0468. \end{aligned} \tag{35}$$

By dividing (35) by (34) we get the (annualized) Sharpe ratio (*S*):

$$S = \frac{R_{annualized} - r_f}{\sigma_{annualized}} = \frac{R_{excess}}{\sigma_{annualized}}\,. \tag{41}$$

The portfolio performances can be summarized in the following table:

[11] Approximately 252 days.

		MDP-short	MDP-long	B	EW
2000-2012	Total period				
	Annualized return	14.91%	14.43%	8.07%	15.04%
	Annualized excess return	10.23%	9.75%	3.39%	10.36%
	Annualized volatility	12.52%	12.46%	24.07%	17.97%
	Sharpe ratio	0.817	0.783	0.141	0.577

Table 1: The returns, volatilities and Sharpe ratios are given in this table for the total period. This is calculated for the MDP-short, MDP-long, the equally-weighted portfolio (EW) and the cap-weighted benchmark (B).

In Table 1, I have calculated the mean of the entire period and multiplied by 252. We see that both MDPs have higher returns and lower standard deviations than the benchmark, hence the higher Sharpe ratios. The equally-weighted portfolio also has a higher Sharpe ratio than the benchmark, but less than the two MDPs.

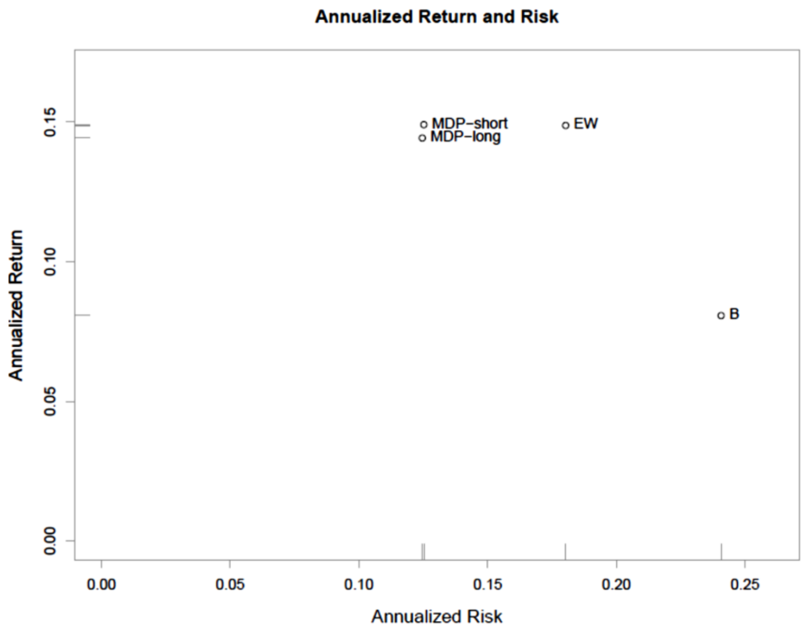

Figure 2 Risk-return scatter plot for the MDP-short, MDP-long, benchmark and the equally-weighted portfolio.

In this figure[12] we clearly see that if we are standing in B we can reduce the volatility and increase the return by moving to the MDPs. If we hold the equally-weighted portfolio, we can lower the volatility by holding one of the MDPs instead. The MDP – long offers slightly

[12] Note that the plot measures the annualized return and not the annualized *excess* return. This does not however affect the relationship between the portfolio performances since the risk-free rate is a constant.

lower annualized return, but has a higher Sharpe ratio. In other words, the benchmark and the equally-weighted portfolio are inefficient compared to the MDPs. Between the two MDPs, the MDP-short is the efficient portfolio. This is, however, a relatively long investment horizon. Therefore, I have calculated the annualized Sharpe ratio for each year in order to see if the MDPs consistently outperform the benchmark. The results are shown in table two.

		2000	2001	2002	2003	2004	2005	2006
MDP-short	Annualized return	9,50%	-7,74%	-7,14%	71,69%	38,56%	52,53%	36,88%
	Annualized excess return	4,82%	-12,42%	-11,82%	67,01%	33,88%	47,85%	32,20%
	Annualized volatility	10,83%	11,15%	14,19%	13,17%	9,71%	11,15%	12,42%
	Sharpe ratio	0,445	-1,113	-0,833	5,090	3,490	4,292	2,592
MDP-long	Annualized return	8,44%	-7,99%	-5,70%	70,38%	38,36%	53,33%	33,08%
	Annualized excess return	3,76%	-12,67%	-10,38%	65,70%	33,68%	48,65%	28,40%
	Annualized volatility	10,61%	11,22%	13,49%	12,72%	9,75%	11,18%	12,04%
	Sharpe ratio	0,354	-1,129	-0,770	5,165	3,455	4,352	2,358
B	Annualized return	15,82%	-10,83%	-29,34%	34,00%	30,81%	35,44%	24,43%
	Annualized excess return	11,14%	-15,51%	-34,02%	29,32%	26,13%	30,76%	19,75%
	Annualized volatility	16,59%	17,83%	20,93%	16,41%	13,93%	15,86%	23,52%
	Sharpe ratio	0,672	-0,870	-1,626	1,786	1,876	1,939	0,840
EW	Annualized return	12,65%	-9,25%	-20,68%	70,84%	44,02%	54,14%	30,16%
	Annualized excess return	7,97%	-13,93%	-25,36%	66,16%	39,34%	49,46%	25,48%
	Annualized volatility	14,17%	14,46%	14,82%	13,28%	12,55%	14,70%	16,21%
	Sharpe ratio	0,562	-0,963	-1,711	4,981	3,135	3,366	1,572

		2007	2008	2009	2010	2011	2012
MDP-short	Annualized return	14,27%	-29,54%	15,06%	7,48%	-21,90%	10,26%
	Annualized excess return	9,59%	-34,22%	10,38%	2,80%	-26,58%	5,58%
	Annualized volatility	10,32%	12,71%	13,00%	12,06%	14,32%	15,17%
	Sharpe ratio	0,930	-2,692	0,798	0,232	-1,856	0,368
MDP-long	Annualized return	15,91%	-36,06%	14,96%	9,27%	-23,23%	13,08%
	Annualized excess return	11,23%	-40,74%	10,28%	4,59%	-27,91%	8,40%
	Annualized volatility	9,91%	14,04%	13,35%	12,32%	13,93%	14,78%
	Sharpe ratio	1,134	-2,903	0,770	0,373	-2,003	0,568
B	Annualized return	10,66%	-75,71%	61,26%	16,26%	-32,68%	21,85%
	Annualized excess return	5,98%	-80,39%	56,58%	11,58%	-37,36%	17,17%
	Annualized volatility	19,36%	46,29%	32,50%	22,26%	27,35%	19,73%
	Sharpe ratio	0,309	-1,737	1,741	0,520	-1,366	0,870
EW	Annualized return	11,02%	-59,29%	44,84%	10,11%	-29,89%	30,43%
	Annualized excess return	6,34%	-63,97%	40,16%	5,43%	-34,57%	25,75%
	Annualized volatility	14,06%	30,21%	18,79%	16,99%	19,90%	24,32%
	Sharpe ratio	0,451	-2,117	2,137	0,320	-1,737	1,059

Table 2 The annualized returns, excess returns, volatilities and Sharpe ratios each year from 2000-2012.

To calculate the annualized return and volatility each year, I have multiplied the daily average return of each year by the number of trading days in a year. When dealing with negative Sharpe ratios, it is important to keep in mind that a portfolio with a Sharpe ratio closer to zero is the efficient portfolio (given that none of the comparable portfolios have positive values). From the table we see that the MDPs do not deliver superior Sharpe ratios each year. This is illustrated more clearly in the following figure.

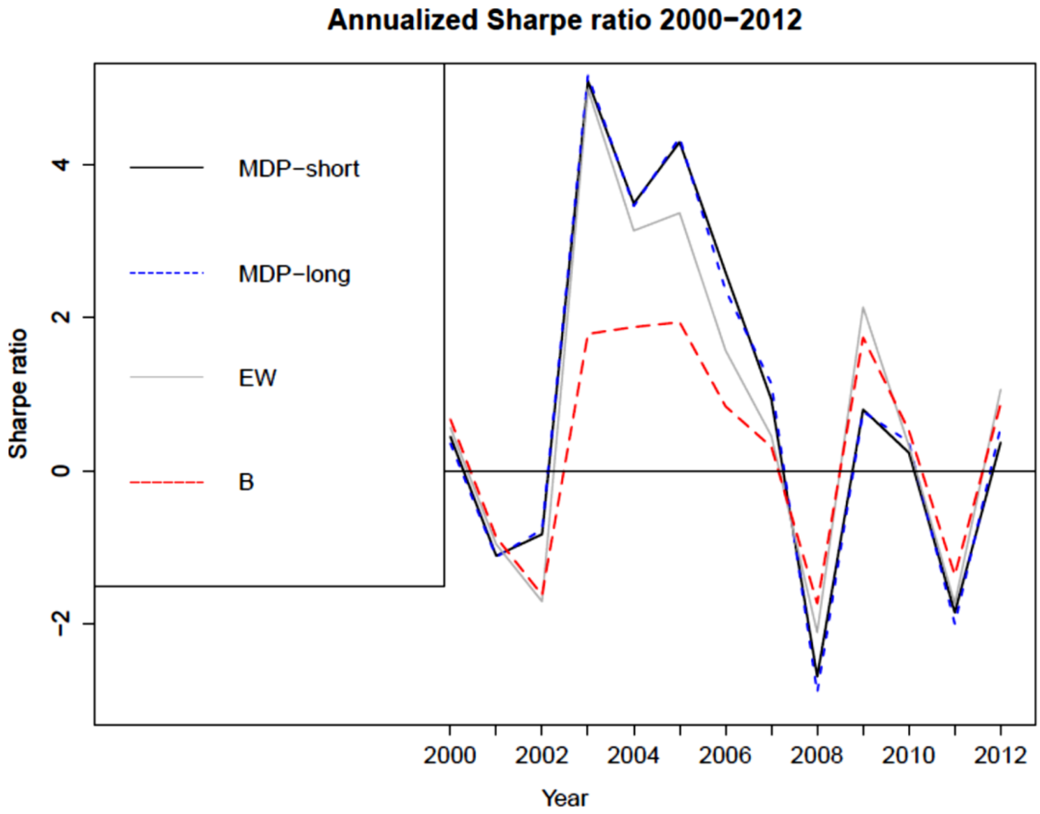

Figure 3 The annualized Sharpe ratios each year for MDP-short, MDP-long, the benchmark and the equally-weighted portfolio.

All four portfolios have decreasing Sharpe ratios in the beginning. This is clearly a reaction to the dot-com bubble that burst March 2000 (Chaffey, 2010). During the 90's IT-based companies experienced a tremendous upswing. Because of the promising prospects of this technology, these companies were overvalued. This dot-com bubble continued to increase and finally burst in the beginning of 2000. A lot of these firms went bankrupt and only a few survived. In the Norwegian stock market there are 6 out of 35 IT-based companies left. The Sharpe ratios start to increase during 2002. All portfolios have their peaks in 2003, except for the benchmark which has its highest value in 2005. In 2008 every portfolio hits rock bottom. This is when the financial crisis hit the global economy. In the rest of the period the Sharpe ratios are fluctuating. We see a new downfall in 2011, which may be due to the following uncertainty in the Eurozone countries, such as Spain, Italy and Greece. The benchmark has the highest Sharpe ratio in the beginning and at the end of the period. In every downfall, the benchmark is the most efficient portfolio. The two MDPs seem to follow each other closely in the beginning. They deliver superior Sharpe ratios from 2002 to 2007. The equally-weighted portfolio tends to lie a bit lower than the two MDPs, but has clearly the highest Sharpe ratio in 2009.

Another way to measure portfolio performances is to compare the cumulative returns, i.e. we look at the future value of investing in the portfolios at the start of the period.

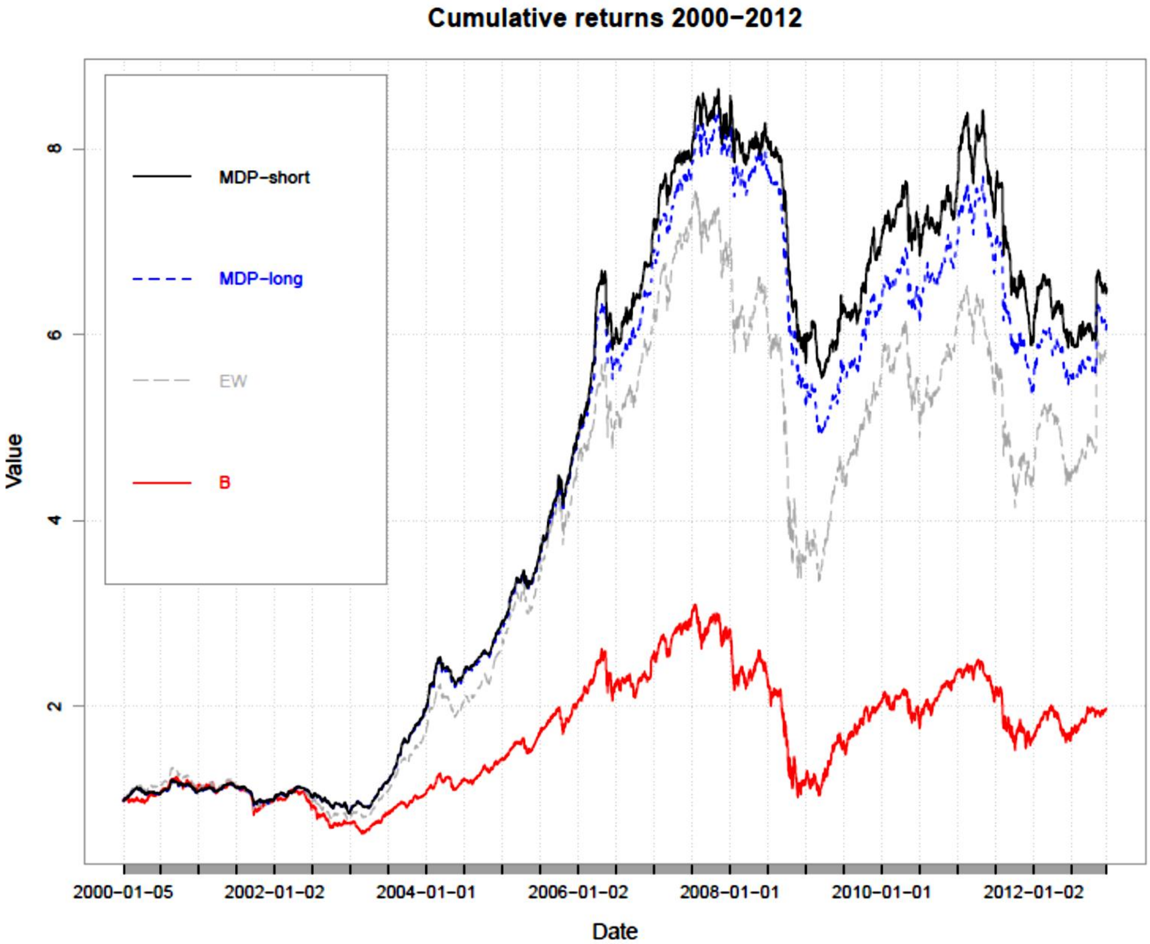

Figure 4: The cumulative returns for the four portfolios in the period 2000-2012. This figure shows the future value of investing $1 at the beginning of the period.

From Figure 4 we see that both MDPs outperform the cap-weighted benchmark. MDP – short has overall the highest cumulative return. This portfolio has its peak in the beginning of 2008. The $1 investment has grown over 8 times its original value, while the investment in the benchmark has grown approximately 3 times its original value. MDP – long follows the MDP – short closely, and has its peak at a little over 8. The equally weighted portfolio has at this point grown to a little over 7.5 times its original value. All four portfolios have a downturn right after their peaks in the beginning of 2008. This, of course, is due to the financial crisis. After the downfall in 2008 the cumulative returns seems to flat out for all portfolios.

5.2 Diversification

5.2.1 Diversification ratio

In order to compare the four portfolios, each observation of the diversification ratio is plotted in the figure below. The diversification ratio is calculated at the end of each quarter from 2000-2012.

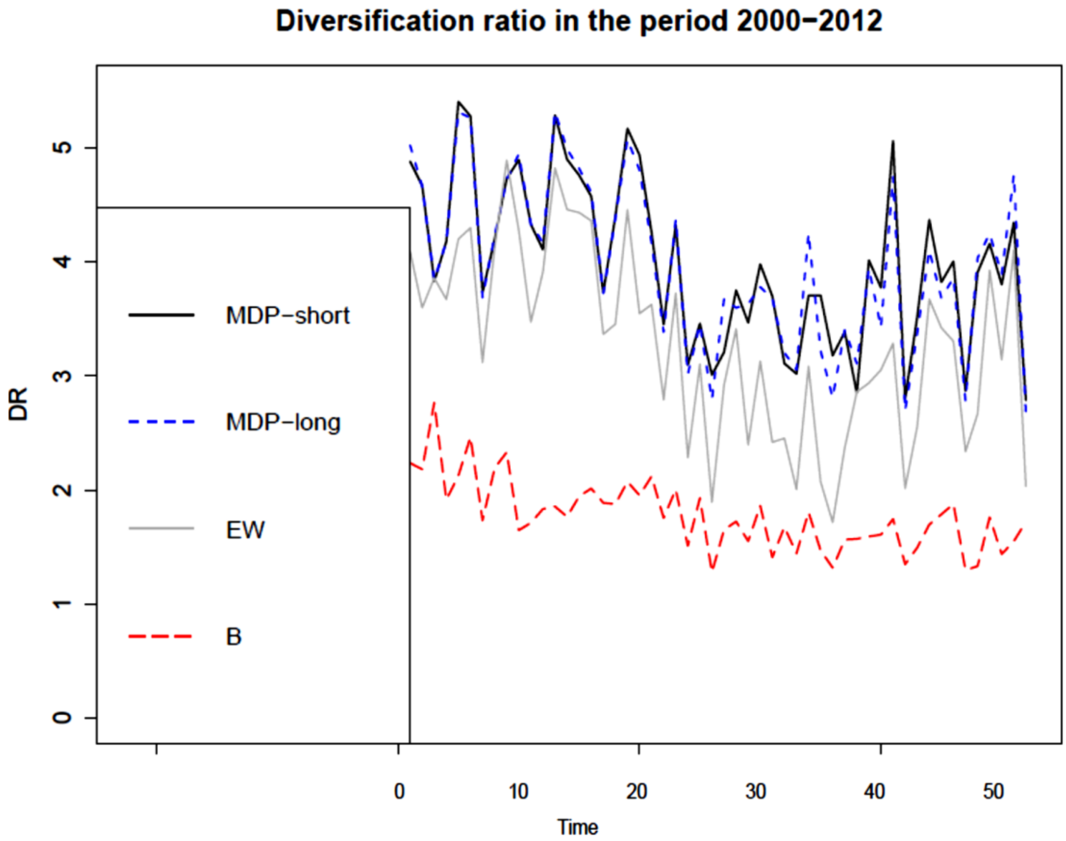

Figure 5: The figure shows the diversification ratio for the four portfolios from 2000-2012 on a quarterly basis, i.e. 1 on the x-axis refers to the 1st quarter of 2000.

The benchmark lies below the two MDPs in the entire period. It is difficult to distinguish between the two most-diversified portfolios by just looking at this plot. They both range from 3 to 5 while the benchmark ranges from 1.5 to almost 3. The equally-weighted portfolio lies a bit lower than the two MDPs. All four portfolios tend to move downwards. This change occur around 25 on the Time-axis, which refers to the second quarter of 2006.

From this we can calculate the average diversification ratio $(\overline{DR})$. This can be defined as:

$$\overline{DR} = \frac{1}{T}\sum_{t=1}^{T} DR_t.$$

In this case $T = 52$, since the diversification ratio is measured on a quarterly basis from 2000-2012. The results are shown in the table below.

Average Diversification Ratio	MDP-short	MDP-long	B	EW
Total period (2000-2012)	4.00	3.98	1.78	3.29

Table 3: The table shows the average diversification ratio for the four portfolios in the total period.

From Table 3 we see that the MDPs have almost identical diversification ratios. This is interesting because when the weights of the MDP - long are constructed, the highest possible ratio it can accomplish would be the same as the MDP – short portfolio[13]. It turns out that the *actual* diversification ratio is approximately the same. The market benchmark has the lowest average DR. Since the equally-weighted portfolio has a small amount of every stock, this could be defined as a well-diversified portfolio. This is also supported by the high diversification ratio relative to the benchmark. It does not however exceed the average diversification ratio of the two MDPs.

5.2.2 Volatility

From Table 1 we saw that the MDP-long had the lowest annualized volatility. This is based on the daily standard deviations. In addition I calculated the average daily risk at the end of every quarter from 2000 to 2012 to compare the risk movements of the four portfolios. This is displayed in the figure below.

[13] In that case there would be no short positions in the MDP – short.

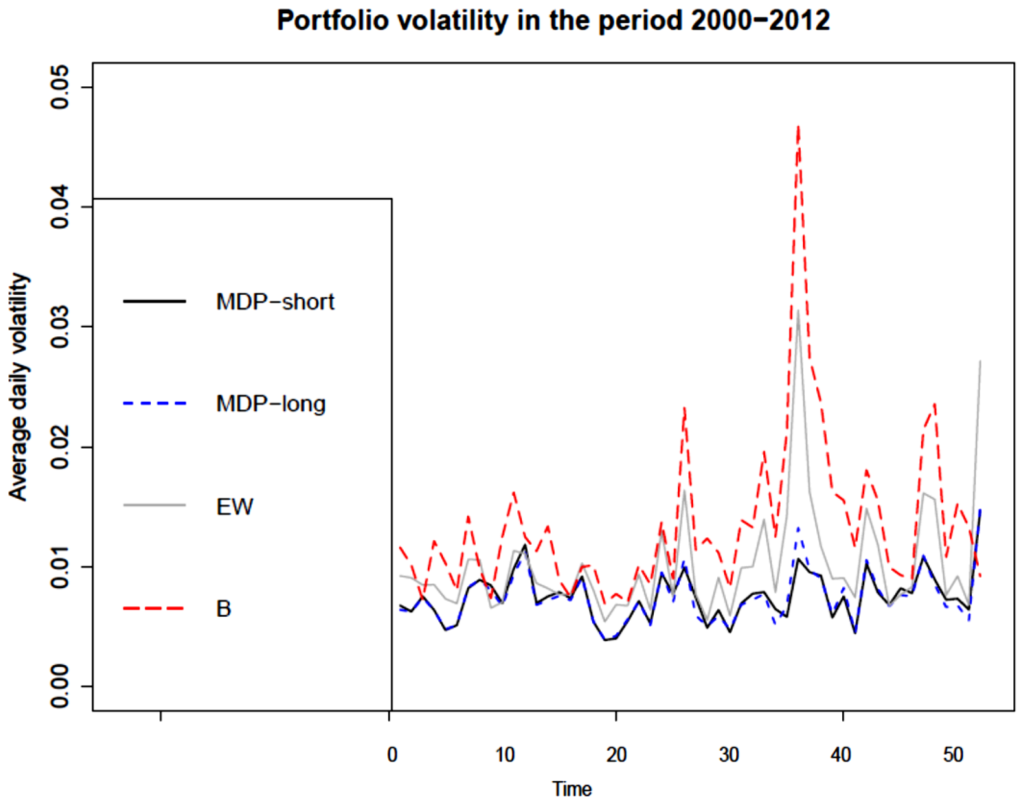

Figure 6: The average daily volatility for each quarter from 2000-2012.

Overall the two MDPs have lower volatilities than the benchmark. In the beginning, the differences are small but increase after the second quarter of 2006 (25 on the Time-axis). In the end of 2008 the difference is at its largest of approximately 4%. When looking at the total period, the standard deviations of the two MDPs have a smaller range than the cap-weighted benchmark. The equally-weighted portfolio seems to follow the benchmark but has consistently lower volatility, except for the last observation. At the end we see that the benchmark is headed downwards, while both MDPs and the equally weighted portfolio are headed upwards.

5.2.3 Correlation

As mentioned in subchapter 4.2.2, the correlation between each stock and the MDP – short is constant. However, this is just the case if we have perfect forecast of the necessary inputs. In other words, since the weights are based on historical data it is not certain that correlation is constant in the performance period. I have calculated the correlation of each stock to the four portfolios. If a portfolio is highly correlated with a majority of stocks, then this could mean that the portfolio is poorly diversified.

	MDP-short	MDP-long	EW	B
ABG SUNDAL CLI HLDG	0.2060	0.2507	0.3350	0.3330
AF GRUPPEN A	0.1749	0.1797	0.1523	0.1049
ARENDALS FOSSEKOMPANI	0.1587	0.1593	0.0805	0.0633
ATEA	0.2291	0.2834	0.4264	0.4112
AVOCET MINING	0.1981	0.2068	0.2333	0.2140
BELSHIPS	0.1946	0.2124	0.2364	0.1751
BLOM	0.2062	0.2243	0.2910	0.2142
BONHEUR	0.2359	0.2952	0.4270	0.3909
BORGESTAD A	0.1943	0.1990	0.1441	0.1128
BYGGMA	0.1875	0.1899	0.0887	0.0470
CONTEXTVISION	0.1940	0.2015	0.1829	0.1223
DATA RESPONS	0.2023	0.2112	0.2507	0.1974
DNB	0.1863	0.2981	0.5453	0.7431
DNO INTERNATIONAL	0.2220	0.2969	0.4969	0.5275
EKORNES	0.1857	0.2058	0.2575	0.2599
EMS SEVEN SEAS	0.2219	0.2270	0.2275	0.1318
FARSTAD SHIPPING	0.2094	0.2390	0.3446	0.3381
FRED OLSEN ENERGY	0.2525	0.3339	0.5242	0.5516
GANGER ROLF	0.2334	0.3112	0.4672	0.4521
GOODTECH	0.2052	0.2143	0.1808	0.1067
GYLDENDAL	0.1736	0.1797	0.0696	0.0429
HAFSLUND A	0.1767	0.2007	0.2474	0.2499
HAFSLUND B	0.1833	0.2255	0.3248	0.3282
HEXAGON COMPOSITES	0.2087	0.2187	0.2539	0.1751
IGE RESOURCES	0.1982	0.2040	0.2733	0.1742
IM SKAUGEN	0.1765	0.1868	0.1725	0.1198
JINHUI SHIP TRSP	0.2227	0.2821	0.4111	0.3687
KITRON	0.2164	0.2285	0.2451	0.2020
KONGSBERG GRUPPEN	0.2011	0.2341	0.3370	0.3612
MARINE HARVEST	0.1923	0.2036	0.2899	0.2354
NAMSOS TRAFIKKSELSKAP	0.1688	0.1746	0.0790	0.0404
NORDIC SEMICONDUCTOR	0.2486	0.2609	0.2651	0.1968
NORSE ENERGY CORP	0.2210	0.2432	0.3702	0.3227
NORSK HYDRO	0.2320	0.3531	0.6036	0.8225
NORSKE SKOGINDUSTRIER	0.1966	0.2490	0.4263	0.4658
NORTHLAND RESOURCES	0.1294	0.1386	0.2230	0.1191
NORWEGIAN CAR CARRIERS	0.1931	0.1958	0.1076	0.0478
ODFJELL A	0.1992	0.2358	0.3081	0.2676
ODFJELL B	0.1987	0.2218	0.2775	0.2246
OLAV THON EIEP	0.1694	0.1904	0.2308	0.2232
ORKLA	0.2027	0.3212	0.5511	0.7114
PETROLEUM GEO	0.2265	0.2978	0.4899	0.5322

SERVICES				
PETROLIA	0.2847	0.3011	0.3921	0.0628
PROSAFE	0.2507	0.3465	0.5878	0.6609
REACH SUBSEA	0.1949	0.1937	0.1948	0.0637
RIEBER SON	0.1347	0.1456	0.1193	0.0935
ROCKSOURCE	0.2087	0.2379	0.2857	0.2030
SAS	0.2118	0.2439	0.3594	0.3875
SCANA INDUSTRIER	0.1938	0.2088	0.2253	0.1606
SCHIBSTED	0.2204	0.2881	0.4763	0.5391
SKIENS AKTIEMOLLE	0.1580	0.1623	0.0917	0.0708
SOLSTAD OFFSHORE	0.2023	0.2331	0.3412	0.3324
SOLVANG	0.1896	0.1951	0.0902	0.0383
SPAREBANK 1 SR BANK	0.1861	0.2250	0.3372	0.3537
STOLT NIELSEN	0.2540	0.3046	0.4425	0.4339
STOREBRAND	0.2061	0.2889	0.5187	0.6345
SUBSEA 7	0.2365	0.3142	0.5212	0.5705
TGS NOPEC GEOPHS	0.2498	0.3488	0.5897	0.6423
TIDE	0.1937	0.2019	0.0722	0.0246
TOMRA SYSTEMS	0.2076	0.2387	0.3550	0.4144
TTS GROUP	0.1930	0.2448	0.3040	0.2598
VEIDEKKE	0.2181	0.2541	0.3573	0.3638
VOSS VEKSEL OG LMDBK	0.1945	0.2001	0.0297	-0.0040
WILHS WILHELMSEN HDG A	0.2109	0.2416	0.3423	0.3210
WILHS WILHELMSEN HDG B	0.2012	0.2187	0.2653	0.2329

Table 4: The correlation between every stock and each of the four portfolios.

From Table 4 we see that the MDP – long, the equally-weighted portfolio and the cap-weighted benchmark have highest correlation coefficients with Norsk Hydro (35.31%, 60.36% and 82.25% respectively). MDP – short has its highest correlation with Petrolia (28.47%). Overall, the MDPs have lower correlations than both the equally-weighted portfolio and the benchmark. This is illustrated more clearly in the following figure.

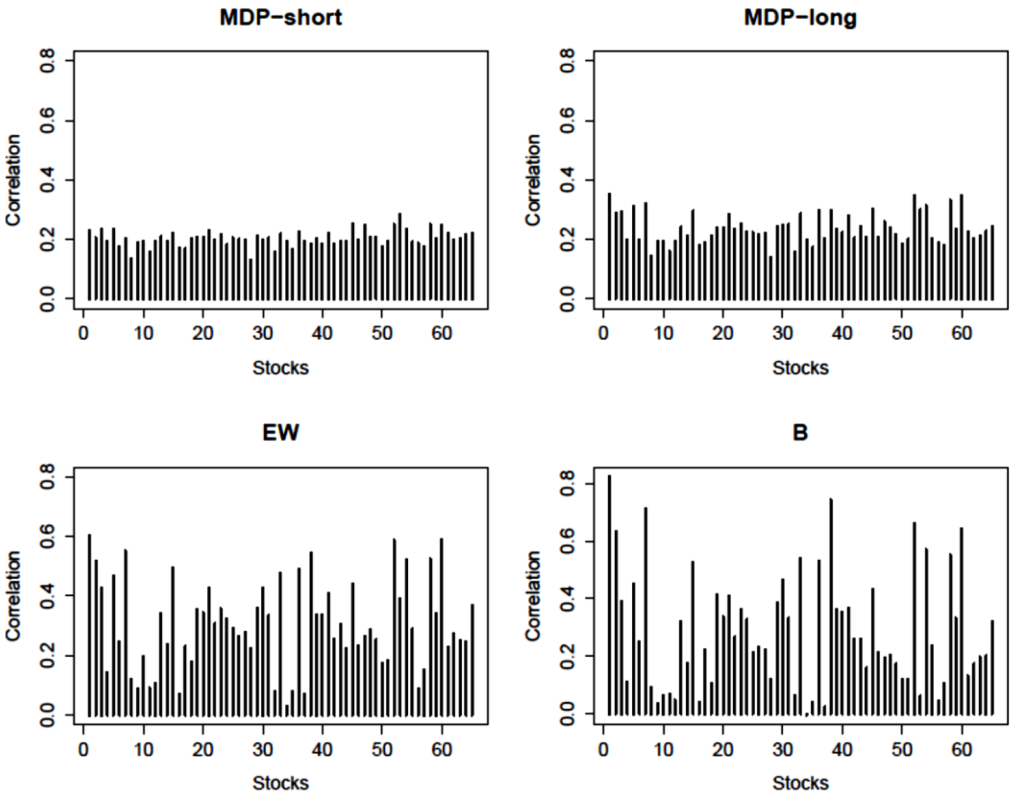

Figure 7: The correlation between each of the 65 stocks and the four portfolios.

Although the correlations are not constant for the MDPs (see chapter 4.2.2), they have a significantly smaller range than the equally-weighted portfolio and the benchmark.

CHAPTER 6 CONCLUSION

In this thesis I have analyzed the performance and diversification of different portfolios in the period 2000-2012. I have introduced the diversification ratio, and maximized this ratio in order to create the most-diversified portfolios. To estimate the covariance matrix, I used a shrinkage approach which is developed by Ledoit & Wolf (2003), and enhanced by Schäfer & Strimmer (2005). I decomposed the diversification ratio in a volatility-weighted average correlation term and a volatility-weighted concentration ratio for a broader understanding of the forces behind it.

I created a capitalization-weighted portfolio and an equally-weighted portfolio. The cumulative returns plot showed that the two MDPs outperform both portfolios. This also holds in terms of Sharpe ratios. Overall, the MDP-short has the highest Sharpe ratio. The cap-weighted portfolio has the lowest. When we look at the annualized Sharpe ratios for each year, none of the portfolios deliver consistently superior performance. The MDPs have the lowest Sharpe ratios in 2001 and 2008-2012, while the capitalization-weighted benchmark has the highest Sharpe ratio in these periods.

The diversification ratios are consistently higher for the two MDPs. From the DR-decomposition we know that this originates from lower average correlation and/or lower concentration ratio. The equally-weighted portfolio has a low concentration on each stock, but has overall a higher correlation to each stock. This lowers the diversification ratio. The MDPs also have lower daily standard deviations in the whole period, except towards the end. The greatest difference between the daily portfolio standard deviations is at the end of 2008.

In addition, I looked at the correlation between each of the 65 stocks and the four portfolios. Overall, the MDPs have lower correlation than the cap-weighted benchmark and the equally-weighted portfolio.

According to the Sharpe ratios and the cumulative returns plot, the MDP – short outperforms the MDP – long. When we look at the portfolio diversification, the intuition would tell us that MDP – short is more diversified than MDP – long, due to the possibility of negative weights. However, in this analysis it is hard to spot major differences in diversification when we look at the diversification ratio and the daily volatility. The correlation to each stock is a bit lower for the MDP – short, although this difference is hard to spot.

APPENDIX A

List of firms used in the analysis:

ABG SUNDAL CLI HLDG
AF GRUPPEN A
ARENDALS FOSSEKOMPANI
ATEA
AVOCET MINING
BELSHIPS
BLOM
BONHEUR
BORGESTAD A
BYGGMA
CONTEXTVISION
DATA RESPONS
DNB
DNO INTERNATIONAL
EKORNES
EMS SEVEN SEAS
FARSTAD SHIPPING
FRED OLSEN ENERGY
GANGER ROLF
GOODTECH
GYLDENDAL
HAFSLUND A
HAFSLUND B
HEXAGON COMPOSITES
IGE RESOURCES
IM SKAUGEN
JINHUI SHIP TRSP
KITRON
KONGSBERG GRUPPEN
MARINE HARVEST
NAMSOS TRAFIKKSELSKAP
NORDIC SEMICONDUCTOR
NORSE ENERGY CORP
NORSK HYDRO
NORSKE SKOGINDUSTRIER
NORTHLAND RESOURCES
NORWEGIAN CAR CARRIERS
ODFJELL A
ODFJELL B

OLAV THON EIEP
ORKLA
PETROLEUM GEO SERVICES
PETROLIA
PROSAFE
REACH SUBSEA
RIEBER SON
ROCKSOURCE
SAS
SCANA INDUSTRIER
SCHIBSTED
SKIENS AKTIEMOLLE
SOLSTAD OFFSHORE
SOLVANG
SPAREBANK 1 SR BANK
STOLT NIELSEN
STOREBRAND
SUBSEA 7
TGS NOPEC GEOPHS
TIDE
TOMRA SYSTEMS
TTS GROUP
VEIDEKKE
VOSS VEKSEL OG LMDBK
WILHS WILHELMSEN HDG A
WILHS WILHELMSEN HDG B

Appendix B

The two approaches in chapter 4.1 lead to the same solution. This can be illustrated as follows:

Define $\tilde{\boldsymbol{\sigma}}$ as a matrix with *n* rows and *n* columns, with the volatility of each stock along the diagonal. The upper – and lower triangles are filled with zeros:

$$\tilde{\boldsymbol{\sigma}} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n \end{pmatrix}.$$

The covariance matrix can be expressed as:

$$\boldsymbol{\Omega} = \tilde{\boldsymbol{\sigma}}\mathbf{C}\tilde{\boldsymbol{\sigma}}$$

where **C** is the correlation matrix.

The inverse covariance matrix can be expressed as:

$$\boldsymbol{\Omega}^{-1} = \tilde{\boldsymbol{\sigma}}^{-1}\mathbf{C}^{-1}\tilde{\boldsymbol{\sigma}}^{-1}.$$

Since $\tilde{\boldsymbol{\sigma}}^{-1} \cdot \boldsymbol{\sigma} = \mathbf{1}$, equation 34.1 can be written as:

$$\frac{\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}}{\boldsymbol{\sigma}^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}} = \tilde{\boldsymbol{\sigma}}^{-1}\frac{\mathbf{C}^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{C}^{-1}\mathbf{1}}.$$

We remember from the second approach that we had to divide each asset weight with its volatility. This has already been accounted for in the expression above, since we multiply by $\tilde{\boldsymbol{\sigma}}^{-1}$. In other words, in the second approach we could have written:

$$\boldsymbol{M} = \begin{pmatrix} \frac{w_{S1}^*}{\sigma_1} \\ \frac{w_{S2}^*}{\sigma_2} \\ \vdots \\ \frac{w_{Sn}^*}{\sigma_n} \end{pmatrix} = \tilde{\boldsymbol{\sigma}}^{-1}\mathbf{w}_s.$$

Finally, we need to rescale the weights to sum to 1.

Appendix C

Below is a list of the scripts and functions I used in R[14].

```
######################Creating MDP-short#########################

library(timeSeries)
library(PerformanceAnalytics)
library(corpcor) #for cov.shrink and invcov.shrink

#Import dataset
price <- read.csv("C:/.../StockPrice.csv", sep=";", dec=",")


#Function that generates the optimal weigths for maximizing div
#ratio. It also calculates the diversification ratio and the
#portfolio volatility.

#Inputs:
#Period1 = historical data
#Period2 = performance data

monthly.MDP = function(Period1,Period2)
{
  call = match.call()

  period1.ts = as.timeSeries(Period1,drop=FALSE)
  period1.ret = returns(period1.ts,method="simple")
  period1.retmat = as.matrix(period1.ret)

  #Inputs to calculate optimal weights
  cov.mat = cov(period1.ret)
  st.dev = cbind(apply(period1.ret,2,sd))
  cov.mat.inv <- invcov.shrink(period1.ret)

  #optimale weights

  top.mat = cov.mat.inv%*%st.dev
  bot.mat = as.numeric(t(st.dev)%*%top.mat)
  weights = top.mat/bot.mat
  weights = as.vector(w.dmax)
  w.dmax = weights/sum(weights)

  #Performance period
  period2.ts = as.timeSeries(Period2,drop=FALSE)
  period2.ret = returns(period2.ts,method="simple")
  period2.retmat = as.matrix(period2.ret)
```

---

[14] I created a function called “monthly.MDP( )”. In the beginning of the process I rebalanced the portfolios on a monthly basis. When I decided to rebalance on a quarterly basis instead, it would take a lot of time to rename the function for every quarter, since I used the same script.

```
  #Calculate the most-diversified portfolio(MDP)
  MDP = period2.retmat%*%w.dmax
  colnames(MDP) = c("MDP-short")
  MDP.ts = as.timeSeries(MDP,drop=FALSE)

  #Calculate diversification ratio
  cov.mat2 = cov(period2.ret)
  st.dev2 = cbind(apply(period2.ret,2,sd))
  DR = (t(w.dmax)%*%st.dev2)/sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)
  port.vol = sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)

  dmax.port = list("call"=call,
                   "MDP" = MDP.ts,
                   "DR" = DR)
  class(dmax.port) = "portfolio"
  dmax.port
}

##########################Creating MDP-long######################
library(timeSeries)
library(PerformanceAnalytics)
library(corpcor)
library(quadprog)

monthly.MDP = function(Period1,Period2)
{
  call = match.call()

  period1.ts = as.timeSeries(Period1,drop=FALSE)
  period1.ret = returns(period1.ts,method="simple")
  period1.retmat = as.matrix(period1.ret)

  cov.mat = cov(period1.ret)
  st.dev = cbind(apply(period1.ret,2,sd))


  #Inputs to quadratic optimization
  Amat = cbind(st.dev, diag(65))
  dvec= rep(0,65)
  Dmat= cov.shrink(period1.ret)
  bvec = c(1,rep(0,65))

  MDPLong = solve.QP(Dmat=Dmat, dvec=dvec, Amat=Amat, bvec=bvec,
meq=1)
  w.dmax = MDPLong$solution/sum(MDPLong$solution)
  w.dmax = round(w.dmax, digits=4)

  #Performance period
  period2.ts = as.timeSeries(Period2,drop=FALSE)
  period2.ret = returns(period2.ts,method="simple")
  period2.retmat = as.matrix(period2.ret)


  #Calculate the most-diversified portfolio(MDP)
  MDP = period2.retmat%*%w.dmax
```

```
  colnames(MDP) = c("MDP-long")
  MDP.ts = as.timeSeries(MDP,drop=FALSE)

  #Calculate diversification ratio
  cov.mat2 = cov(period2.ret)
  st.dev2 = cbind(apply(period2.ret,2,sd))
  DR = (t(w.dmax)%*%st.dev2)/sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)
  port.vol = sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)

  dmax.port = list("call"=call,
                   "MDP" = MDP.ts,
                   "DR" = DR)
  class(dmax.port) = "portfolio"
  dmax.port
}

########################Combine each quarter#################

#We need to apply these functions every quarter from 2000-2012.
#Run the function for MDP-short

MDP1 = monthly.MDP(price[1:262,],price[263:327,])
MDP2 = monthly.MDP(price[65:327,],price[328:392,])
MDP3 = monthly.MDP(price[129:392,],price[393:457,])
MDP4 = monthly.MDP(price[193:457,],price[458:522,])
MDP5 = monthly.MDP(price[257:522,],price[523:587,])
MDP6 = monthly.MDP(price[321:587,],price[588:652,])
MDP7 = monthly.MDP(price[385:652,], price[653:717,])
MDP8 = monthly.MDP(price[449:717,], price[718:782,])
MDP9 = monthly.MDP(price[513:782,],price[783:847,])
MDP10 = monthly.MDP(price[577:847,],price[848:912,])
MDP11 = monthly.MDP(price[641:912,],price[913:977,])
MDP12 = monthly.MDP(price[705:977,],price[978:1042,])
MDP13 = monthly.MDP(price[769:1042,], price[1043:1107,])
MDP14 = monthly.MDP(price[833:1107,],price[1108:1172,])
MDP15 = monthly.MDP(price[897:1172,], price[1173:1237,])
MDP16 = monthly.MDP(price[961:1237,], price[1238:1302,])
MDP17 = monthly.MDP(price[1025:1302,],price[1303:1367,])
MDP18 = monthly.MDP(price[1089:1367,], price[1368:1432,])
MDP19 = monthly.MDP(price[1153:1432,], price[1433:1497,])
MDP20 = monthly.MDP(price[1217:1497,], price[1498:1562,])
MDP21 = monthly.MDP(price[1281:1562,], price[1563:1627,])
MDP22 = monthly.MDP(price[1345:1627,], price[1628:1692,])
MDP23 = monthly.MDP(price[1409:1692,], price[1693:1757,])
MDP24 = monthly.MDP(price[1473:1757,], price[1758:1822,])
MDP25 = monthly.MDP(price[1573:1822,], price[1823:1887,])
MDP26 = monthly.MDP(price[1637:1887,], price[1888:1952,])
MDP27 = monthly.MDP(price[1701:1952,], price[1953:2017,])
MDP28 = monthly.MDP(price[1765:2017,], price[2018:2082,])
MDP29 = monthly.MDP(price[1829:2082,], price[2083:2147,])
MDP30 = monthly.MDP(price[1893:2147,],price[2148:2212,])
MDP31 = monthly.MDP(price[1957:2212,], price[2213:2277,])
MDP32 = monthly.MDP(price[2021:2277,], price[2278:2342,])
MDP33 = monthly.MDP(price[2085:2342,], price[2343:2407,])
MDP34 = monthly.MDP(price[2149:2407,], price[2408:2472,])
```

```
MDP35 = monthly.MDP(price[2213:2472,], price[2473:2537,])
MDP36 = monthly.MDP(price[2277:2537,], price[2538:2602,])
MDP37 = monthly.MDP(price[2341:2602,], price[2603:2667,])
MDP38 = monthly.MDP(price[2405:2667,], price[2668:2732,])
MDP39 = monthly.MDP(price[2469:2732,], price[2733:2797,])
MDP40 = monthly.MDP(price[2533:2797,], price[2798:2862,])
MDP41 = monthly.MDP(price[2597:2862,], price[2863:2927,])
MDP42 = monthly.MDP(price[2661:2927,], price[2928:2992,])
MDP43 = monthly.MDP(price[2725:2992,], price[2993:3057,])
MDP44 = monthly.MDP(price[2789:3057,], price[3058:3122,])
MDP45 = monthly.MDP(price[2853:3122,], price[3123:3187,])
MDP46 = monthly.MDP(price[2917:3187,], price[3188:3252,])
MDP47 = monthly.MDP(price[2981:3252,], price[3253:3317,])
MDP48 = monthly.MDP(price[3045:3317,], price[3318:3382,])
MDP49 = monthly.MDP(price[3109:3382,], price[3383:3447,])
MDP50 = monthly.MDP(price[3173:3447,], price[3448:3512,])
MDP51 = monthly.MDP(price[3237:3512,], price[3513:3577,])
MDP52 = monthly.MDP(price[3301:3577,], price[3578:3642,])

#MDP-short
total.mdp=
rbind(MDP1$MDP,MDP2$MDP,MDP3$MDP,MDP4$MDP,MDP5$MDP,MDP6$MDP,MDP7$MDP
,MDP8$MDP,MDP9$MDP,MDP10$MDP,MDP11$MDP,MDP12$MDP,MDP13$MDP,MDP14$MDP
,MDP15$MDP,MDP16$MDP,MDP17$MDP,MDP18$MDP,MDP19$MDP,MDP20$MDP,MDP21$M
DP,MDP22$MDP,MDP23$MDP,MDP24$MDP,MDP25$MDP,MDP26$MDP,MDP27$MDP,MDP28
$MDP,MDP29$MDP,MDP30$MDP,MDP31$MDP,MDP32$MDP,MDP33$MDP,MDP34$MDP,MDP
35$MDP,MDP36$MDP,MDP37$MDP,MDP38$MDP,MDP39$MDP,MDP40$MDP,MDP41$MDP,M
DP42$MDP,MDP43$MDP,MDP44$MDP,MDP45$MDP,MDP46$MDP,MDP47$MDP,MDP48$MDP
,MDP49$MDP,MDP50$MDP,MDP51$MDP,MDP52$MDP)

colnames(total.mdp) = c("MDP-short")

#DR
DRMDPS =
rbind(MDP1$DR,MDP2$DR,MDP3$DR,MDP4$DR,MDP5$DR,MDP6$DR,MDP7$DR,MDP8$D
R,MDP9$DR,MDP10$DR,MDP11$DR,MDP12$DR,MDP13$DR,MDP14$DR,MDP15$DR,MDP1
6$DR,MDP17$DR,MDP18$DR,MDP19$DR,MDP20$DR,MDP21$DR,MDP22$DR,MDP23$DR,
MDP24$DR,MDP25$DR,MDP26$DR,MDP27$DR,MDP28$DR,MDP29$DR,MDP30$DR,MDP31
$DR,MDP32$DR,MDP33$DR,MDP34$DR,MDP35$DR,MDP36$DR,MDP37$DR,MDP38$DR,M
DP39$DR,MDP40$DR,MDP41$DR,MDP42$DR,MDP43$DR,MDP44$DR,MDP45$DR,MDP46$
DR,MDP47$DR,MDP48$DR,MDP49$DR,MDP50$DR,MDP51$DR,MDP52$DR)

colnames(DRMDPS) = c("MDP - short")

#Now run the function for MDP-long and re-run this script

#MDP-long
total.mdpLong=
rbind(MDP1$MDP,MDP2$MDP,MDP3$MDP,MDP4$MDP,MDP5$MDP,MDP6$MDP,MDP7$MDP
,MDP8$MDP,MDP9$MDP,MDP10$MDP,MDP11$MDP,MDP12$MDP,MDP13$MDP,MDP14$MDP
,MDP15$MDP,MDP16$MDP,MDP17$MDP,MDP18$MDP,MDP19$MDP,MDP20$MDP,MDP21$M
DP,MDP22$MDP,MDP23$MDP,MDP24$MDP,MDP25$MDP,MDP26$MDP,MDP27$MDP,MDP28
$MDP,MDP29$MDP,MDP30$MDP,MDP31$MDP,MDP32$MDP,MDP33$MDP,MDP34$MDP,MDP
35$MDP,MDP36$MDP,MDP37$MDP,MDP38$MDP,MDP39$MDP,MDP40$MDP,MDP41$MDP,M
```

```
DP42$MDP,MDP43$MDP,MDP44$MDP,MDP45$MDP,MDP46$MDP,MDP47$MDP,MDP48$MDP
,MDP49$MDP,MDP50$MDP,MDP51$MDP,MDP52$MDP)

colnames(total.mdpLong) = c("MDP-long")

#DR
DRMDPL =
rbind(MDP1$DR,MDP2$DR,MDP3$DR,MDP4$DR,MDP5$DR,MDP6$DR,MDP7$DR,MDP8$D
R,MDP9$DR,MDP10$DR,MDP11$DR,MDP12$DR,MDP13$DR,MDP14$DR,MDP15$DR,MDP1
6$DR,MDP17$DR,MDP18$DR,MDP19$DR,MDP20$DR,MDP21$DR,MDP22$DR,MDP23$DR,
MDP24$DR,MDP25$DR,MDP26$DR,MDP27$DR,MDP28$DR,MDP29$DR,MDP30$DR,MDP31
$DR,MDP32$DR,MDP33$DR,MDP34$DR,MDP35$DR,MDP36$DR,MDP37$DR,MDP38$DR,M
DP39$DR,MDP40$DR,MDP41$DR,MDP42$DR,MDP43$DR,MDP44$DR,MDP45$DR,MDP46$
DR,MDP47$DR,MDP48$DR,MDP49$DR,MDP50$DR,MDP51$DR,MDP52$DR)

colnames(DRMDPS) = c("MDP-long")


###################Creating the cap-weighted benchmark#############
library(PerformanceAnalytics)
library(timeSeries)

#Import dataset
#Stock prices and weights
price <- read.csv("C:/.../StockPrice.csv", sep=";", dec=",")
weights <- read.csv("C:/.../weights.csv", sep=";", dec=",")

#Function for benchmark
#Inputs:
#1. w.dmax = a column vector of weights (nx1)
#2. Period2 = performance period

cap.benchmark = function(w.dmax,Period2)
{
  call = match.call()

  #Performance period
  period2.ts = as.timeSeries(Period2,drop=FALSE)
  period2.ret = returns(period2.ts,method="simple")
  period2.retmat = as.matrix(period2.ret)


  #Calculate MDP-long
  MDP = period2.retmat%*%w.dmax
  colnames(MDP) = c("MDP-long")
  MDP.ts = as.timeSeries(MDP,drop=FALSE)

  #Calculate diversification rate and portfolio volatility
  cov.mat2 = cov(period2.ret)
  st.dev2 = cbind(apply(period2.ret,2,sd))
  DR = (t(w.dmax)%*%st.dev2)/sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)
  port.vol = sqrt(t(w.dmax)%*%cov.mat2%*%w.dmax)

  dmax.port = list("call"=call,
                   "MDP" = MDP.ts,
```

```
                  "DR" = DR)
  class(dmax.port) = "portfolio"
  dmax.port
}

#Calculating portfolio returns for each quarter
MDP1 = cap.benchmark (weights[,2],price[263:327,])
MDP2 = cap.benchmark (weights[,3],price[328:392,])
MDP3 = cap.benchmark (weights[,4],price[393:457,])
MDP4 = cap.benchmark (weights[,5],price[458:522,])
MDP5 = cap.benchmark (weights[,6],price[523:587,])
MDP6 = cap.benchmark (weights[,7],price[588:652,])
MDP7 = cap.benchmark (weights[,8], price[653:717,])
MDP8 = cap.benchmark (weights[,9], price[718:782,])
MDP9 = cap.benchmark (weights[,10],price[783:847,])
MDP10 = cap.benchmark (weights[,11],price[848:912,])
MDP11 = cap.benchmark (weights[,12],price[913:977,])
MDP12 = cap.benchmark (weights[,13],price[978:1042,])
MDP13 = cap.benchmark (weights[,14], price[1043:1107,])
MDP14 = cap.benchmark (weights[,15],price[1108:1172,])
MDP15 = cap.benchmark (weights[,16], price[1173:1237,])
MDP16 = cap.benchmark (weights[,17], price[1238:1302,])
MDP17 = cap.benchmark (weights[,18],price[1303:1367,])
MDP18 = cap.benchmark (weights[,19], price[1368:1432,])
MDP19 = cap.benchmark (weights[,20], price[1433:1497,])
MDP20 = cap.benchmark (weights[,21], price[1498:1562,])
MDP21 = cap.benchmark (weights[,22], price[1563:1627,])
MDP22 = cap.benchmark (weights[,23], price[1628:1692,])
MDP23 = cap.benchmark (weights[,24], price[1693:1757,])
MDP24 = cap.benchmark (weights[,25], price[1758:1822,])
MDP25 = cap.benchmark (weights[,26], price[1823:1887,])
MDP26 = cap.benchmark (weights[,27], price[1888:1952,])
MDP27 = cap.benchmark (weights[,28], price[1953:2017,])
MDP28 = cap.benchmark (weights[,29], price[2018:2082,])
MDP29 = cap.benchmark (weights[,30], price[2083:2147,])
MDP30 = cap.benchmark (weights[,31],price[2148:2212,])
MDP31 = cap.benchmark (weights[,32], price[2213:2277,])
MDP32 = cap.benchmark (weights[,33], price[2278:2342,])
MDP33 = cap.benchmark (weights[,34], price[2343:2407,])
MDP34 = cap.benchmark (weights[,35], price[2408:2472,])
MDP35 = cap.benchmark (weights[,36], price[2473:2537,])
MDP36 = cap.benchmark (weights[,37], price[2538:2602,])
MDP37 = cap.benchmark (weights[,38], price[2603:2667,])
MDP38 = cap.benchmark (weights[,39], price[2668:2732,])
MDP39 = cap.benchmark (weights[,40], price[2733:2797,])
MDP40 = cap.benchmark (weights[,41], price[2798:2862,])
MDP41 = cap.benchmark (weights[,42], price[2863:2927,])
MDP42 = cap.benchmark (weights[,43], price[2928:2992,])
MDP43 = cap.benchmark (weights[,44], price[2993:3057,])
MDP44 = cap.benchmark (weights[,45], price[3058:3122,])
MDP45 = cap.benchmark (weights[,46], price[3123:3187,])
MDP46 = cap.benchmark (weights[,47], price[3188:3252,])
MDP47 = cap.benchmark (weights[,48], price[3253:3317,])
MDP48 = cap.benchmark (weights[,49], price[3318:3382,])
MDP49 = cap.benchmark (weights[,50], price[3383:3447,])
```

```
MDP50 = cap.benchmark (weights[,51], price[3448:3512,])
MDP51 = cap.benchmark (weights[,52], price[3513:3577,])
MDP52 = cap.benchmark (weights[,53], price[3578:3642,])

#Benchmark
total.bench=
rbind(MDP1$MDP,MDP2$MDP,MDP3$MDP,MDP4$MDP,MDP5$MDP,MDP6$MDP,MDP7$MDP
,MDP8$MDP,MDP9$MDP,MDP10$MDP,MDP11$MDP,MDP12$MDP,MDP13$MDP,MDP14$MDP
,MDP15$MDP,MDP16$MDP,MDP17$MDP,MDP18$MDP,MDP19$MDP,MDP20$MDP,MDP21$M
DP,MDP22$MDP,MDP23$MDP,MDP24$MDP,MDP25$MDP,MDP26$MDP,MDP27$MDP,MDP28
$MDP,MDP29$MDP,MDP30$MDP,MDP31$MDP,MDP32$MDP,MDP33$MDP,MDP34$MDP,MDP
35$MDP,MDP36$MDP,MDP37$MDP,MDP38$MDP,MDP39$MDP,MDP40$MDP,MDP41$MDP,M
DP42$MDP,MDP43$MDP,MDP44$MDP,MDP45$MDP,MDP46$MDP,MDP47$MDP,MDP48$MDP
,MDP49$MDP,MDP50$MDP,MDP51$MDP,MDP52$MDP)

colnames(total.bench) = c("B")

#DR
DRBench =
rbind(MDP1$DR,MDP2$DR,MDP3$DR,MDP4$DR,MDP5$DR,MDP6$DR,MDP7$DR,MDP8$D
R,MDP9$DR,MDP10$DR,MDP11$DR,MDP12$DR,MDP13$DR,MDP14$DR,MDP15$DR,MDP1
6$DR,MDP17$DR,MDP18$DR,MDP19$DR,MDP20$DR,MDP21$DR,MDP22$DR,MDP23$DR,
MDP24$DR,MDP25$DR,MDP26$DR,MDP27$DR,MDP28$DR,MDP29$DR,MDP30$DR,MDP31
$DR,MDP32$DR,MDP33$DR,MDP34$DR,MDP35$DR,MDP36$DR,MDP37$DR,MDP38$DR,M
DP39$DR,MDP40$DR,MDP41$DR,MDP42$DR,MDP43$DR,MDP44$DR,MDP45$DR,MDP46$
DR,MDP47$DR,MDP48$DR,MDP49$DR,MDP50$DR,MDP51$DR,MDP52$DR)


#############Creating the Equally-weighted portfolio###########
library(PerformanceAnalytics)
library(timeSeries)

#Import dataset
price <- read.csv("C:/.../StockPrice.csv", sep=";", dec=",")

weights = rep(1/65,65)

#Performance period
perf.ts = as.timeSeries(price[263:3642,],drop="FALSE")
perf.ret = returns(perf.ts, method="simple")
perf.retmat = as.matrix(perf.ret)

#Since the MDPs and the benchmark are rebalanced each quarter, we
#loose an observation each time. These observations must also be
#removed for EW, otherwise there are specific functions we can't
#use to compare the portfolios.

#Create a vector that contains the row numbers which will be
#removed
RemoveRow = c(65, 130, 195, 260, 325, 390, 455, 520, 585, 650, 715,
780,845, 910, 975, 1040, 1105, 1170, 1235, 1300, 1365, 1430,
1495,1560, 1625, 1690, 1755, 1820, 1885, 1950, 2015, 2080, 2145,
2210,2275, 2340, 2405, 2470, 2535, 2600, 2665, 2730, 2795, 2860,
2925,2990, 3055, 3120, 3185, 3250, 3315)
perf.retmat = perf.retmat[-(RemoveRow),]
```

```
eq.port = perf.retmat%*%weights

colnames(eq.port) = "EW"
eq.ts = as.timeSeries(eq.port, drop=FALSE)

#I used the function cap.benchmark to calculate the
#diversification ratio and portfolio volatility for EW by
#substituting the equal weights by w.dmax


#################Plots##################
library(PerformanceAnalytics)
library(timeSeries)

#Combine all four portfolios
total.Merge = cbind(total.mdp, total.mdpLong, eq.ts, total.bench)

#Risk and return
chart.RiskReturnScatter(total.Merge, sharpe.ratio=NULL)

#Cumulative returns plot
chart.CumReturns(total.Merge, wealth.index=TRUE,lwd=2,
legend.loc="topleft", main="Cumulative returns in the period 2000-
2012")

#DR
plot(DRMDPS, type="l", lwd=2,col="black", ylim=c(1,5.5),xlim=c(-
20,52), main="Diversification ratio 2000-2012", ylab="Div ratio",
xlab="Time")
lines(DRMDPL, lty="dashed",lwd=2, col="blue")
lines(DREW, lty="solid", lwd=2,col="grey")
lines(DRBench, lty="longdash",lwd=2, col="red")
legend(x="topleft", col=c("black","blue","grey","red"),
lty=c("solid","dashed","solid","longdash"),lwd=2)

#Portfolio volatility
plot(MDPS.vol,lwd=2, type="l", col="black", ylim=c(0,0.05),
ylab="Volatility", main="Portfolio volatility in the period 2000-
2012", xlim=c(-20,52),xlab="Time")
lines(MDPL.vol,lwd=2, lty="dashed", col="blue")
lines(EW.vol,lwd=2, lty="solid", col="grey")
lines(Bench.vol, lwd=2,lty="longdash", col="red")
legend(x="bottomleft", col=c("black","blue","grey","red"),
lty=c("solid","dashed","solid","longdash"),lwd=2)

#DR against volatility
plot(DRMDPS, MDPS.vol, ylab="Volatility", xlab="Div ratio",
main="MDP-short")
plot(DRMDPL,MDPL.vol,ylab="Volatility", xlab="Div ratio", main="MDP-
long")
plot(DREW,EW.vol, ylab="Volatility", xlab="Div ratio", main="EW")
plot(DRBench, Bench.vol, ylab="Volatility", xlab="Div ratio",

main="B")
```

```
#################Correlation####################
library(PerformanceAnalytics)
library(timeSeries)


correlation.Portf = function(Stock,MDPS,MDPL,EW,Bench)
{
  call=match.call()
  Stock.ts = as.timeSeries(Stock,drop=FALSE)
  Merged = cbind(Stock.ts,MDPS,MDPL,EW,Bench)
  cor.portf = cor(Merged)

  dmax.port = list("call"=call,
                   "Cor" = cor.portf[,1]
                   )
  class(dmax.port) = "portfolio"
  dmax.port

}

cor.matrix = matrix(nrow=65,ncol=4)

cor1 =
correlation.Portf(perf.retmat[,1],total.mdp,total.mdpLong,eq.ts,tota
l.bench)
cor.matrix[1,]=cor1$Cor[2:5]

#Replace cor1 by cor2 and cor.matrix[1,] by cor.matrix[2,]
#all the way to 65.

round(cor.matrix, digits =4)

#Plot correlations
par(mfrow=c(2,2))
plot(cor.matrix[,1],type="h",lwd=2,ylab="Correlation",
xlab="Stocks", main="MDP-short",ylim=c(0,0.8))
plot(cor.matrix[,2],type="h",lwd=2,ylab="Correlation",
xlab="Stocks", main="MDP-long",ylim=c(0,0.8))
plot(cor.matrix[,3],type="h",lwd=2,ylab="Correlation",
xlab="Stocks", main="EW",ylim=c(0,0.8))
plot(cor.matrix[,4],type="h",lwd=2,ylab="Correlation",
xlab="Stocks", main="B")


#######################Shrinkage example##################
library(gdata) #lowerTriangle()
library(PerformanceAnalytics)
library(corpcor)

#Returns
StockA = rbind(0.02,0.01,0.05,0.025)
StockB = rbind(0.01,0.07,-0.005,0.1)
StockC = rbind(0.1,0.09,0.12,0.13)
Merged = cbind(StockA,StockB,StockC)
```

```
######The variance shrinkage intensity

xA.bar = (sum(StockA))/4
xB.bar = sum(StockB)/4
xC.bar = sum(StockC)/4

w.A1=  (StockA[1,]-xA.bar)^2
w.A2 = (StockA[2,]-xA.bar)^2
w.A3 = (StockA[3,]-xA.bar)^2
w.A4 = (StockA[4,]-xA.bar)^2
w.B1 = (StockB[1,]-xB.bar)^2
w.B2 = (StockB[2,]-xB.bar)^2
w.B3 = (StockB[3,]-xB.bar)^2
w.B4 = (StockB[4,]-xB.bar)^2
w.C1 = (StockC[1,]-xC.bar)^2
w.C2 = (StockC[2,]-xC.bar)^2
w.C3 = (StockC[3,]-xC.bar)^2
w.C4 = (StockC[4,]-xC.bar)^2

wbar_A = sum(w.A1,w.A2,w.A3,w.A4)/4
wbar_B = sum(w.B1,w.B2,w.B3,w.B4)/4
wbar_C = sum(w.C1,w.C2,w.C3,w.C4)/4

vA = wbar_A*(4/3)
vB = wbar_B*(4/3)
vC = wbar_C*(4/3)

var.vA = sum((w.A1-wbar_A)^2,(w.A2-wbar_A)^2,(w.A3-wbar_A)^2,(w.A4-
wbar_A)^2)*(4/(3^3))
var.vB = sum((w.B1-wbar_B)^2,(w.B2-wbar_B)^2,(w.B3-wbar_B)^2,(w.B4-
wbar_B)^2)*(4/(3^3))
var.vC = sum((w.C1-wbar_C)^2,(w.C2-wbar_C)^2,(w.C3-wbar_C)^2,(w.C4-
wbar_C)^2)*(4/(3^3))

var.vec = diag(var(Merged))
median.var = median(var.vec)

#Now computing lambda
l.top = sum(var.vA,var.vB,var.vC)
l.bot = sum((vA-median.var)^2,(vB-median.var)^2,(vC-median.var)^2)
lambda.var = l.top/l.bot

lambda.var #0.1363
#When using the estimate.lambda.var function in corpcor
estimate.lambda.var(Merged) #0.1363

#Now calculating the estimated variance and volatility
vol.shrink = sqrt((lambda.var*median.var)+(1-lambda.var*var.vec))
vol.shrink.mat = matrix(nrow = 3,ncol=3)
diag(vol.shrink.mat)= vol.shrink

##########The correlation shrinkage intensity

#Volatilities
st.devA = apply(StockA,2,sd)
```

```
st.devB = apply(StockB,2,sd)
st.devC = apply(StockC,2,sd)

w.k1ij1= ((StockA[1,]-xA.bar)*(StockB[1,]-xB.bar))/(st.devA*st.devB)
w.k1ij2 =((StockA[1,]-xA.bar) *(StockC[1,]-
xC.bar))/(st.devA*st.devC)
w.k1ij3 = ((StockB[1,]-xB.bar)*(StockC[1,]-
xC.bar))/(st.devB*st.devC)
w.k2ij1 = ((StockA[2,]-xA.bar)*(StockB[2,]-
xB.bar))/(st.devA*st.devB)
w.k2ij2 = ((StockA[2,]-xA.bar)*(StockC[2,]-
xC.bar))/(st.devA*st.devC)
w.k2ij3= ((StockB[2,]-xB.bar)*(StockC[2,]-xC.bar))/(st.devB*st.devC)
w.k3ij1 = ((StockA[3,]-xA.bar)*(StockB[3,]-
xB.bar))/(st.devA*st.devB)
w.k3ij2 = ((StockA[3,]-xA.bar)*(StockC[3,]-
xC.bar))/(st.devA*st.devC)
w.k3ij3 = ((StockB[3,]-xB.bar)*(StockC[3,]-
xC.bar))/(st.devB*st.devC)
w.k4ij1 = ((StockA[4,]-xA.bar)*(StockB[4,]-
xB.bar))/(st.devA*st.devB)
w.k4ij2 = ((StockA[4,]-xA.bar)*(StockC[4,]-
xC.bar))/(st.devA*st.devC)
w.k4ij3 = ((StockB[4,]-xB.bar)*(StockC[4,]-
xC.bar))/(st.devB*st.devC)

w.barAB = (w.k1ij1+w.k2ij1+w.k3ij1+w.k4ij1)/4
w.barAC = (w.k1ij2+w.k2ij2+w.k3ij2+w.k4ij2)/4
w.barBC = (w.k1ij3+w.k2ij3+w.k3ij3+w.k4ij3)/4

top1 = (w.k1ij1-w.barAB)^2+(w.k2ij1-w.barAB)^2+(w.k3ij1-
w.barAB)^2+(w.k4ij1-w.barAB)^2
top2 = (w.k1ij2-w.barAC)^2+(w.k2ij2-w.barAC)^2+(w.k3ij2-
w.barAC)^2+(w.k4ij2-w.barAC)^2
top3 = (w.k1ij3-w.barBC)^2+(w.k2ij3-w.barBC)^2+(w.k3ij3-
w.barBC)^2+(w.k4ij3-w.barBC)^2
top.tot = (4/(3^3))*(top1+top2+top3)

cor.sq = lowerTriangle(cor(Merged)^2)

lambda.cor = top.tot/sum(cor.sq)
lambda.cor #0.8726

#When using the estimate.lambda function in corpcor
estimate.lambda(Merged) #0.8726

#Estimated correlation matrix
identity.mat = diag(x=1, nrow=3, ncol=3)
cor.shrink = lambda.cor*identity.mat+(1-lambda.cor)*cor(Merged)

#The estimated covariance matrix
covmat.shrink = vol.shrink.mat%*%cor.shrink%*%vol.shrink.mat

#By using the corpcor package
cov.matshrink1 = cov.shrink(Merged)
```

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